

## Quantum Horizons of the standard model landscape

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**ABSTRACT:** The long-distance effective field theory of our Universe — the Standard Model coupled to gravity — has a unique 4D vacuum, but we show that it also has a landscape of lower-dimensional vacua, with the potential for moduli arising from vacuum and Casimir energies. For minimal Majorana neutrino masses, we find a near-continuous infinity of  $\text{AdS}_3 \times S_1$  vacua, with circumference  $\sim 20$  microns and  $\text{AdS}_3$  length  $4 \times 10^{25}$  m. By AdS/CFT, there is a  $\text{CFT}_2$  of central charge  $c \sim 10^{90}$  which contains the Standard Model (and beyond) coupled to quantum gravity in this vacuum. Physics in these vacua is the same as in ours for energies between  $10^{-1}$  eV and  $10^{48}$  GeV, so this  $\text{CFT}_2$  also describes all the physics of our vacuum in this energy range. We show that it is possible to realize quantum-stabilized AdS vacua as near-horizon regions of new kinds of quantum extremal black objects in the higher-dimensional space — near critical black strings in 4D, near-critical black holes in 3D. The violation of the null-energy condition by the Casimir energy is crucial for these horizons to exist, as has already been realized for analogous non-extremal 3D black holes by Emparan, Fabbri and Kaloper. The new extremal 3D black holes are particularly interesting — they are (meta)stable with an entropy independent of  $\hbar$  and  $G_N$ , so a microscopic counting of the entropy may be possible in the  $G_N \rightarrow 0$  limit. Our results suggest that it should be possible to realize the larger landscape of AdS vacua in string theory as near-horizon geometries of new extremal black brane solutions.

**KEYWORDS:** Neutrino Physics, Black Holes, AdS-CFT Correspondence, Standard Model.

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## 1. Preamble

M-theory is a unique theory with a unique 11-dimensional vacuum. However it also has an enormous landscape of lower-dimensional vacua, which raises the thorny questions of vacuum selection. The long distance effective theory of our world — the Standard Model coupled to gravity — is an effective field theory in 4 dimensions, with some fixed microphysics, and also has a unique 4D vacuum. In this paper, we begin by showing that there is also a Standard Model landscape, by exhibiting a near-continuous infinity of lower-dimensional vacua of the theory. The simplest example is compactification on a circle, where the potential for the radius modulus receives competing contributions from the tiny cosmological constant, as well as the Casimir energies of the graviton, photon and, crucially, the massive neutrinos. With Majorana neutrinos, whose masses are constrained by

explaining the atmospheric and solar neutrino anomalies, we find an  $\text{AdS}_3 \times S_1$  vacuum of the theory with the circumference of the  $S_1$  at about  $\sim 20$  microns. With Dirac neutrinos, both  $\text{AdS}_3$  as well as  $dS_3$  vacua are possible. Lower-dimensional vacua can exist as well. These solutions exist completely independently of any UV completion of the theory at the electroweak scale and beyond. Of course if string theory is correct and our 4D vacuum is part of the theory, then the vacua we are describing are part of the string landscape as well. While we focus on the Standard Model landscape here, such vacua would seem to be generic in non-supersymmetric theories where the cosmological constant is fine-tuned to be small.

The  $\text{AdS}_3$  vacua are particularly interesting: it is often thought that AdS/CFT can not be used to describe quantum gravity in our world because we have a positive cosmological constant. But this is not the case in these  $\text{AdS}_3 \times S_1$  vacua! By AdS/CFT, there must be some two-dimensional conformal field theory description of this background. Since the size of the  $S_1$  is so large, all of conventional high-energy physics — the spectrum of leptons and hadrons, electroweak symmetry breaking, whatever completes the Standard Model up to the Planck scale, even very high energy scattering probing quantum gravity at energies well above the Planck scale but beneath energies that would make a  $\sim 20$  micron sized black-hole — is the same in this vacuum as in ours. Of course we can't yet identify this CFT, but its existence as the dual description of quantum gravity in a *very* close cousin of our world is quite interesting.

After discussing the vacua, we turn to the interesting question of what physical processes can connect or interpolate between them. We will see that there are novel extremal black holes and black strings which asymptote to the 4D vacua and realize the lower-dimensional AdS vacua as their near-horizon geometries, in a way analogous to ordinary extremal charged black holes and branes that interpolate from Minkowski space to  $\text{AdS}_m \times S_n$  vacua. What is interesting is that these are intrinsically *quantum* black objects — such horizons can not exist classically due to familiar no-hair arguments which follow from an energy momentum tensor satisfying the null energy condition. However the energy conditions are violated by the Casimir energies, which play the crucial role in modulus stabilization to begin with. Schwarzschild-type non-extremal quantum black holes supported by Casimir energy have been studied recently by Emparan, Fabbri and Kaloper [1]; our further contribution here is (A) to realize that these objects exist as solutions in the Standard Model and (B) to place them in a broader context, revealing also the extremal black holes and their role as interpolators in the Standard Model landscape. These novel sorts of black hole are very interesting and we will discuss a number of their properties. We will also discuss some interpolations to the lower-dimensional dS vacua as well.

In the simplest case of the Standard Model  $\text{AdS}_3 \times S_1$  vacuum the interpolating solutions are cosmic strings. Smallness of the Casimir potential implies that the opening angle is very small, so that such cosmic string cannot be present in the visible part of the Universe. However, given that we live in de Sitter space, there is a (tiny) non-zero probability for a dS thermal fluctuation resulting in the creation of this object within our causal patch. Note that this transition does not change the microscopic structure of the vacuum at distances smaller than 20 microns, so that small enough observers — for instance, many

of the *Amoebosoa*—are able to survive it and enter the lower-dimensional vacua.

It is interesting that the presence of some “negative” gravitational energy, violating the null-energy condition, is a crucial part of *all* realistic modulus stabilization mechanisms; in string theory a common source of the negative energies come from the negative-tension orientifold planes, while in our Standard Model vacua it arises from Casimir energy. It is natural to conjecture that *all* the AdS vacua in the larger string landscape can be thought of as the near-horizon limits of “exotic” extremal black holes in 10 dimensions, with the no-hair theorems being evaded by the negative energies needed for modulus stabilization. If true, it would be interesting to probe these vacua from the “outside”.

## 2. The standard model landscape

We will now show that the action of the minimal Standard Model (SM) plus General Relativity (GR) has more than one distinct vacuum, actually a true landscape of vacua. Let us start considering the SM+GR action compactified on a circle of radius  $R$ . At distances larger than  $R$ , there is an effective 3D theory with a metric parameterized by

$$ds_{(4)}^2 = \frac{r^2}{R^2} ds_{(3)}^2 + R^2 \left( d\phi - \frac{\sqrt{2}}{M_4 r} V_\mu dx^\mu \right)^2, \quad (2.1)$$

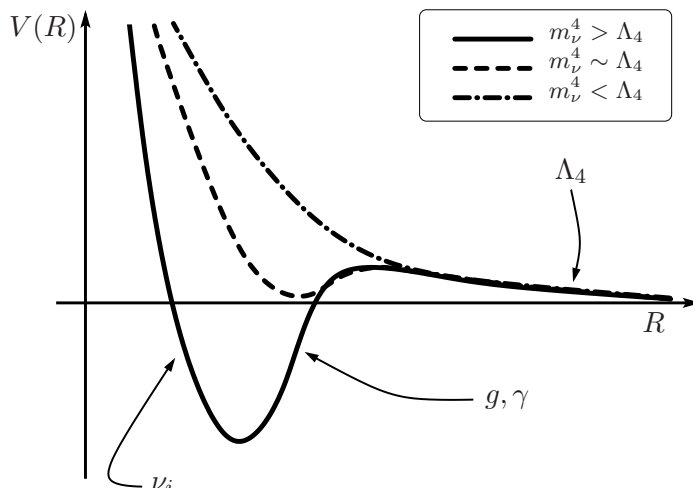
where  $M_4$  is the 4D reduced Planck mass ( $1/\sqrt{8\pi G_N}$ ),  $R$  is the radion field,  $V_\mu$  is the graviphoton,  $\phi \in [0, 2\pi)$ , and  $r$  is an arbitrary scale that we will later fix to the expectation value of  $R$ . With such parameterization the effective action is already in the Einstein frame, in particular, the reduction of the action for the pure gravitational sector reads

$$\begin{aligned} S_{\text{grav}} &= \int d^3x d\phi \sqrt{-g_{(4)}} \left( \frac{1}{2} M_4^2 \mathcal{R}_{(4)} - \Lambda_4 \right) \\ &\rightarrow \int d^3x \sqrt{-g_{(3)}} (2\pi r) \left[ \frac{1}{2} M_4^2 \mathcal{R}_{(3)} - \frac{1}{4} \frac{R^4}{r^4} V_{\mu\nu} V^{\mu\nu} - M_4^2 \left( \frac{\partial R}{R} \right)^2 - \frac{r^2 \Lambda_4}{R^2} \right], \end{aligned}$$

where  $\Lambda_4$  is the 4D cosmological constant and  $V_{\mu\nu}$  is the field strength of the graviphoton. Because of the 4D cosmological constant, the classical potential for the radion is *runaway*, which makes the circle decompactify. Indeed this rolling solution is just the expanding 4D de Sitter solution.

However, the smallness of the cosmological constant and the absence of other classical contributions to the effective potential for the radion make quantum corrections important for the study of the stabilization of the compact dimension. The 1-loop corrections to the radion potential is the Casimir energy coming from loops wrapping the circle, which are UV insensitive and calculable. The Casimir potential for a particle of mass  $m$  is  $\propto e^{-2\pi m R}$  for  $R \gg 1/m$ , so at any scale  $R$ , only particles with mass lighter than  $1/R$  are relevant. The contribution to the effective potential of a massless state (with periodic boundary conditions) is

$$\mp \frac{n_0}{720\pi} \frac{r^3}{R^6}, \quad (2.2)$$



**Figure 1:** Radion potential around the neutrino-cosmological constant scale. The regions where the cosmological constant, the massless bosons and the neutrino contributions dominate are indicated with arrows. Depending on the neutrino spectrum the three plots show the three possible scenarios: no vacua, dS and AdS vacuum.

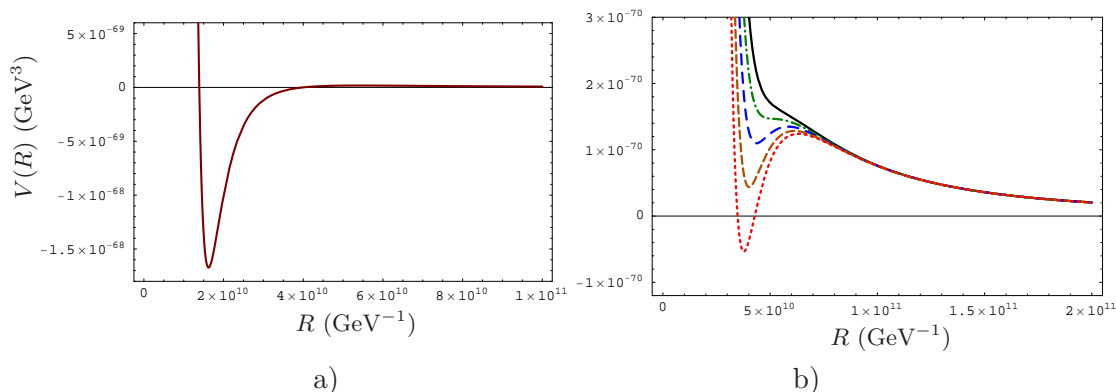
where the sign  $\mp$  is for bosons/fermions and  $n_0$  is the number of degrees of freedom (see appendix A for details). The only massless particles (we know of!) in the SM are the graviton and the photon. For very large radii the cosmological constant contribution wins and the radion potential is runaway while for small radii the Casimir force wins and the compact dimension start shrinking. We thus get a maximum for  $R = R_{\text{max}}$ , with

$$R_{\text{max}} = \left( \frac{1}{120\pi^2\Lambda_4} \right)^{\frac{1}{4}}, \tag{2.3}$$

where we put  $n_0 = 4$  in eq. (2.2) (2 from the graviton + 2 from the photon) and which, for the current value of the cosmological constant  $\Lambda_4 \simeq 3.25 \cdot 10^{-47} \text{ GeV}^4$  [2], means  $R_{\text{max}} \simeq 14$  microns.

If we start with a size  $R$  smaller than this critical value, the circle wants to shrink, however, when the inverse size becomes comparable to the lightest massive particle, its contribution to the effective potential is not suppressed anymore and can change the behavior of the potential. This is indeed what happens when  $1/R$  approaches the neutrino mass scale. The contribution of fermions to the Casimir energy is indeed opposite to that of bosons and since the neutrino d.o.f. are at least 6 (for Majorana neutrino, 12 for Dirac) at shorter scales their contribution eventually wins against that of bosons. Thus a local minimum in general appears. However, since neutrino masses are of the same order as the scale (2.3), the actual existence of the minimum can depend on the details of the neutrino mass spectrum (see figure 1).

On  $S_1$  there is a discrete choice for the spin connection, which results in the choice of periodic or antiperiodic boundary conditions for fermions. In the first case the contribution has opposite sign with respect to that of bosons, while in the second case is the same. In



**Figure 2:** Effective potential as a function of the radion field  $R$ , in the case of a) Majorana and b) Dirac neutrinos for different choices of the lightest neutrino mass:  $m_{\nu_1}=0$  for Majorana,  $m_{\nu_1}=6.5$  (continuous black), 7.0 (dash-dotted green), 7.5 (long dashed blue), 8.0 (short dashed brown) and 8.5 (dotted red) meV, for Dirac neutrinos. The other neutrino masses (with normal hierarchy) have been fixed with the current values for the mass splittings of eq. (2.4). In the plot the scale  $r$  has been chosen so that  $2\pi r=1 \text{ GeV}^{-1}$ .

order to have a minimum we thus need to impose periodic boundary conditions for the neutrinos and have no more than 3 light fermionic d.o.f., where light here means lighter than the scale of eq. (2.3). If these conditions are not met, the positive contributions from the neutrinos start overwhelming the bosonic ones before the latter are able to develop a maximum, and no minimum is developed as well.

We do not know yet the actual neutrino spectrum, nor whether neutrinos are Majorana or Dirac particles. We only know the mass splittings for solar and atmospheric neutrino oscillations [2],

$$\begin{aligned} \Delta m_{\text{atm}}^2 &\simeq (1.9 \div 3.0) \cdot 10^{-3} \text{ eV}^2, \\ \Delta m_{\odot}^2 &\simeq (8.0 \pm 0.5) \cdot 10^{-5} \text{ eV}^2. \end{aligned} \tag{2.4}$$

If we call  $\nu_i$ , with  $i = 1 \dots 3$ , the  $i$ -th mass eigenstate, such that  $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$ , we have two possibilities: (a) the normal hierarchy spectrum with  $\Delta m_{12}^2 = \Delta m_{\odot}^2$ ,  $\Delta m_{23}^2 = \Delta m_{\text{atm}}^2$ , (b) the inverted hierarchy spectrum with  $\Delta m_{12}^2 = \Delta m_{\text{atm}}^2$ ,  $\Delta m_{23}^2 = \Delta m_{\odot}^2$ . From eq. (2.4) it follows that even assuming  $m_{\nu_1} = 0$ , independently of the hierarchy structure of the neutrino mass spectrum,  $m_{\nu_2} \gtrsim 9 \cdot 10^{-3} \text{ eV}$ .

If neutrinos are Majorana particle, this means that no more than 2 d.o.f. can be lighter than  $1/(2\pi R_{\text{max}})$ . In this case we have necessarily a new Standard Model vacuum! The effective potential at this minimum is always negative (figure 2a), therefore this vacuum solution is  $\text{AdS}_3 \times \text{S}_1$ . The radion at the minimum ( $R_0$ ) is of order  $1/m_{\nu}$ , while both the  $\text{AdS}_3$  length ( $\ell_3$ ) and the radion mass ( $m_R$ ) are of order of the 4D Hubble scale ( $\ell_4$ ). Just to give some numbers, if, for example, we take  $m_{\nu_1} = 0$ ,  $m_{\nu_2}^2 = \Delta m_{\odot}^2$  and  $m_{\nu_3}^2 = \Delta m_{\text{atm}}^2$ ,

we have

$$\begin{aligned}
 R_0 &\simeq 3.2 \mu\text{m}, \\
 \ell_3 &\simeq \ell_4/4 \simeq 3.7 \cdot 10^{25} \text{ m}, \\
 m_R &\simeq 6.5/\ell_3 \simeq 3.5 \cdot 10^{-41} \text{ GeV}.
 \end{aligned}$$

If on the other hand, neutrinos are Dirac, then from eq. (2.4) we get an AdS<sub>3</sub> minimum only if the lighter neutrino mass  $m_{\nu_1}$  is larger than  $\approx 8.3 \cdot 10^{-3}$  eV (normal hierarchy) or  $\approx 3.1 \cdot 10^{-3}$  eV (inverted hierarchy), a metastable dS<sub>3</sub> minimum if  $m_{\nu_1} \approx (7.1 \div 8.3) \cdot 10^{-3}$  eV (normal hierarchy) or  $m_{\nu_1} \approx (2.5 \div 3.1) \cdot 10^{-3}$  eV (inverted hierarchy), and no stationary point if  $m_{\nu_1} \lesssim 7.1 \cdot 10^{-3}$  eV (normal hierarchy) or  $m_{\nu_1} \lesssim 2.5 \cdot 10^{-3}$  eV (inverted hierarchy), see figure 2b.

Depending on the neutrino vacua we can thus have a 3D vacuum with positive, zero or negative cosmological constant. In either case the natural value for the effective vacuum energy will be

$$\Lambda_3 \sim m_\nu^3 \approx \Lambda_4 R_0. \tag{2.5}$$

In the case of positive  $\Lambda_3$  we have a 3D dS vacuum. It is interesting to compare the entropy  $S_3$  associated to the dS<sub>3</sub> horizon with the 4D one ( $S_4$ ). We thus have

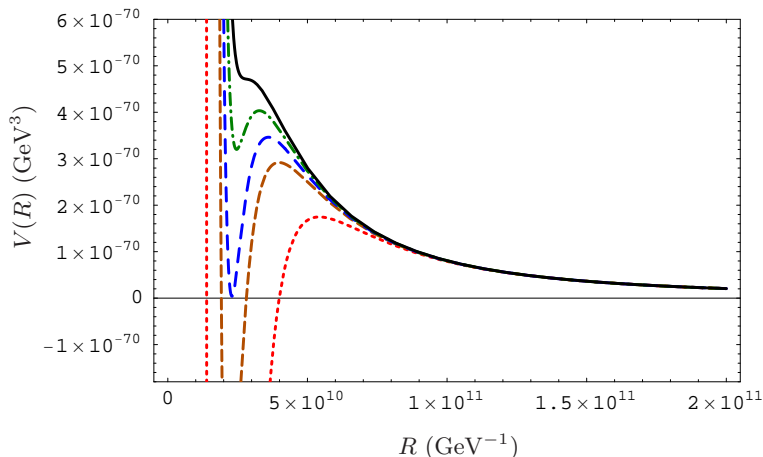
$$S_3 = \frac{M_3}{H_3} \sim M_4^3 R_0^3 \approx \frac{m_\nu}{M_4} S_4,$$

which is much smaller than the 4D dS entropy. In principle, one could also have  $S_3 > S_4$ , since in the limit  $\Lambda_3 \rightarrow 0$   $S_3 \rightarrow \infty$ , however, one would need  $\Lambda_3$  to be suppressed with respect to its natural value in eq. (2.5) by a factor of  $m_\nu^2/M_4^2$ , which turns into a  $10^{-60}$  tuning on the neutrino masses.

Let us stress again that the above analysis depends entirely on IR physics and is independent of UV details, indeed the first non vanishing corrections at one loop would come from the electron (the next lightest state) wrapping around the circle; this contribution is suppressed by  $e^{-2\pi m_e R_0} \sim e^{-m_e/m_\nu}$ ! The calculation is also stable with respect to higher loop corrections, which are small as long as the 4D couplings are perturbative.

The presence and the properties of the neutrino vacuum are very sensitive to both the value of the cosmological constant and the neutrino spectra (see figure 2 and 3). If the cosmological constant had been natural, of order of the Planck or some other high scale, quantum effects would have been negligible at low energies and would have not been able to produce any vacuum. Indeed, just increasing the value of the cosmological constant by an order of magnitude, would be enough to eliminate the presence of the 3D vacuum. Analogously, as shown in figure 3, it is enough to decrease the mass of the second lightest neutrino just by a factor of 3, which means a factor 9 in  $\Delta m_\odot^2$ , to destroy the Majorana-neutrino vacuum (for normal hierarchy).

Stabilization of higher dimensional vacua by Casimir energy has been often considered in the literature in a variety of models, starting from ref. [3]. It is amusing that the Standard Model itself seems to have the right low energy spectrum to allow for stable lower dimensional vacua.



**Figure 3:** Sensitivity of the Majorana neutrino vacuum to  $\Delta m_{\odot}^2$ . The plot shows the radion potential for different values of  $\Delta m_{12}^2 = \Delta m_{\odot}^2$ :  $\Delta m_{\odot}^2 = 8.0$  (dotted red, current value), 2.0 (short dashed brown), 1.5 (long dashed blue), 1.2 (dot-dashed green) and 1.0 (continuous black)  $\cdot 10^{-5}$  eV. The lightest neutrino mass has been fixed to zero. In the plot the scale  $r$  has been chosen so that  $2\pi r = 1 \text{ GeV}^{-1}$ .

### 2.1 A near moduli space

Besides the radion there is another modulus in the compactified 3D action: the longitudinal polarization of the photon  $A_{\phi}$ . Classically, because of gauge invariance, this field is massless. However, at the quantum level, the mass of this field in general gets corrections that depend on the gauge invariant (Wilson loop) combination:

$$W = \exp\left(i \oint_{S_1} A\right).$$

These corrections are generated at one loop by charged fields wrapping  $S_1$ . They can be easily calculated by noticing that, via a gauge transformation, the Wilson loop can be reabsorbed into a non-trivial boundary condition for the charged field. Since a change of the boundary condition will change the contribution of a field to the energy density, this produces a non-trivial potential also for  $A_{\phi}$  (see e.g. [4]). The explicit formula for the potential for  $R$  and  $A_{\phi}$  can be found in the appendix A (eq. (B.1)).

In general charged fermions want to stabilize the Wilson loop around  $A_{\phi} = 1/2$  while charged bosons around  $A_{\phi} = 0$ . In our case the first contribution comes from the electron, it produces a cosine-like potential that stabilizes the Wilson loop around  $A_{\phi} = 1/2$ . There are thus two stationary points, a minimum around  $A_{\phi} = 1/2$  and a maximum around  $A_{\phi} = 0$ . According to [5] both are stable in AdS.

However, because the electron is very heavy compared to our scale, its contribution is exponentially suppressed by a factor of order  $e^{-m_e/m_{\nu}} \sim e^{-10^8}$ ! Because of the smallness of this contribution one may worry whether other would-be subleading corrections may be important. For instance, at higher loop order there are contributions to the effective action with the photon going around the loop. Such corrections are power like and, although



subleading in  $\alpha_{em}$ , might nevertheless be important. It is easy to show, however, that such corrections do not generate a potential for  $A_\phi$  at any order in perturbation theory. Indeed, as long as  $R \gg m_e^{-1}$ , we can integrate out the electron and use the Euler-Heisenberg effective action. In this effective theory there are no minimally coupled particles, all fields are gauge invariant so that the photon possesses an exact shift symmetry that protect it from mass terms at all order in the energy expansion. The perturbative expansion is accurate up to non-perturbative corrections in  $E/m_e$  of order  $e^{-m_e/E} \sim e^{-2\pi m_e R_0}$ , which is of the same order of the contribution from the electron.

Therefore the potential for  $A_\phi$  is effectively flat, in the sense that starting from any value of  $W$  it would take an exponentially long time (say in the  $AdS_3$  length units) to move to the minimum. In this sense the neutrino-vacuum is not unique, effectively there is a continuum of distinct vacua labeled by different values of  $W$ . Strikingly enough we see that the Standard Model, although non-supersymmetric, possesses a near moduli space. The phenomenon that a unique action may give rise to an infinite number of vacua is not a special feature of Superstring/SUSY theories, it is also a feature of the minimal Standard Model!

## 2.2 More vacua

Our analysis in the previous sections was restricted to the simplest Standard Model on a micron sized circle. However, it is natural to expect that there are more vacua. For instance, for smaller size of the circle, more SM states start contributing to the Casimir energy, when bosons and fermions contributions compensate each other, the radion potential can develop a stationary point. The analysis is reported in appendix B.1; apart for a saddle point at the electron scale no new stationary point is present until  $R \sim \Lambda_{QCD}^{-1}$ . The study of the radion potential around the QCD scale would require a non-perturbative analysis. Above this scale, the theory becomes perturbative again. We give the general formula for the effective potential in appendix B.1 but we do not attempt to address the stabilization problem since now the structure of the potential is complicated by the presence of more Wilson loop moduli from gluons and at still shorter distances from electroweak bosons. Extensions of the Standard Model can also affect the Casimir potential, creating new vacua or removing the existing ones. For example the presence of light bosonic fields, like the QCD axion, or extra-dimensional light moduli may favor the presence of the micron vacuum in the case of Dirac neutrinos, while very light fermions, like goldstinos, gravitinos or sterile neutrinos, would tend to destroy such vacuum. Another example is supersymmetry at the TeV scale, which would even the number of bosonic and fermionic d.o.f. and give room to the presence of new vacua at that scale. We comment on some of these possibilities in the appendix B.2.

Another possibility is to compactify more than one dimension. We summarize here the main features of such lower-dimensional vacua, and refer the reader to the appendix (B.3 and B.4) for a detailed analysis. If we compactify two of the spatial dimensions, at low energies the system is well described by a 2D effective theory containing gravity and a set of scalar fields that parameterize the overall size and shape of the manifold we are compactifying on. For instance if we compactify on a two-torus, beside gravity the 2D theory contains the area field  $A$  and the complex modulus  $\tau = \tau_1 + i\tau_2$ . The analysis of this

system is subtler than in the usual case of toroidal compactifications in higher-dimensional models, for in our low-dimensional setups several degrees of freedom are not dynamical. Gravity itself is not dynamical in 2D, neither is the area  $A$ . Their equations of motion are constraint equations that fix, respectively, the total energy and the two-dimensional curvature. More precisely, if there is a two-dimensional potential energy density  $V(A, \tau)$  coming from the 4D cosmological constant, the Casimir energy of light 4D fields, and possibly other sources, then the 2D vacua are characterized by a vanishing potential  $V = 0$  and a curvature  $\mathcal{R}_{(2)} = \partial_A V$ . On the other hand  $\tau$  is dynamical, so in order for a 2D vacuum to be stable it should correspond to a minimum of the potential  $V$  along the  $\tau_{1,2}$  directions. We did not attempt a detailed analysis of the 2D potential in the Standard Model in order to find configurations  $(A_0, \tau_0)$  meeting the above conditions.

The ultimate possibility is compactifying all three spatial dimensions. The resulting theory is a 1D effective theory — quantum mechanics. At low energies the degrees of freedom are the overall size of the compact manifold  $a(t)$  and the shape moduli which we collectively denote by  $\Phi(t)$ . The system is described by a mechanical Lagrangian

$$\mathcal{L} = \frac{1}{2} M_4^2 [ -6 \dot{a}^2 a + a^3 \dot{\Phi} \cdot K(\Phi) \dot{\Phi} ] - V(a, \Phi), \tag{2.6}$$

supplemented by the constraint that the total Hamiltonian vanishes,  $\mathcal{H} = 0$ , the so-called Hamiltonian constraint. In the Lagrangian above  $K(\Phi)$  is a positive definite matrix, while  $a(t)$  enters with a negative definite kinetic energy. Notice that in all previous cases by ‘vacua’ we meant compactified solutions with maximal symmetry (de Sitter, Minkowski, or Anti-de Sitter) in lower dimensions, whereas here in the 1D theory all “fields” only depend on time, and the only sensible definition of a vacuum seems to be ‘a time-independent solution’. However the Hamiltonian constraint makes it impossible for such a solution to exist, unless a perfect tuning is realized in the potential— $V$  should have a stationary point at which  $V$  itself exactly vanishes. Indeed we are used to the fact that the Lagrangian above generically describes a cosmology, the Hamiltonian constraint being just the first Friedman equation. In appendix B.4 we discuss the closest analogue we can have to a vacuum — an almost static micron-sized universe that undergoes classical small oscillations in size and shape on a time-scale of order of our Hubble time. However on longer time-scales such a system is necessarily unstable against decompactification, crunching, or asymmetric Kasner-like evolution, due to the wrong-sign kinetic energy of the scale factor  $a$ .

### 3. AdS/CFT and the real world

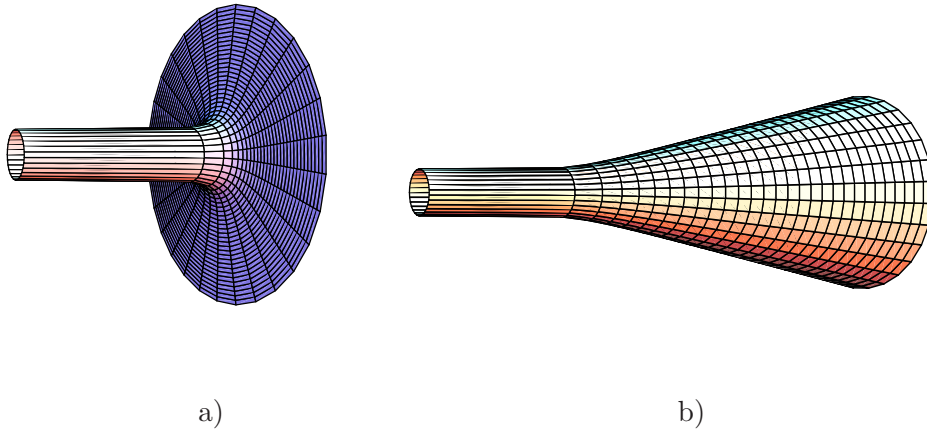
We have seen that, with the minimal particle content consistent with neutrino masses, the Standard Model has  $\text{AdS}_3 \times S_1$  vacua, even though the 4D cosmological constant is small and positive. This vacuum is clearly a very close cousin of our own — since the size of the circle is  $\sim 20$  microns, the high-energy physics in this vacuum — including the Standard Model spectrum, whatever UV completes it all the way up to the Planck scale, even trans-Planckian quantum gravitational physics up to energies up to  $10^{48}$  GeV where  $\sim 20$  micron black holes are produced — is the same as in ours.

By AdS/CFT duality [6] there must exist a two-dimensional CFT dual to physics in this background. Of course this must be a very peculiar CFT. The central charge is  $c \sim \ell_3 M_3 \sim 10^{90}$ . The spectrum of operator dimensions is strange — there are a few operators with  $O(1)$  dimensions, dual to the metric, the photon, the graviphoton and the radion. The operator dual to the Wilson line is rather bizarre — it is nearly marginal, with an anomalous dimension of order  $e^{-10^8}$ ! There is an enormous gap till the operators dual to neutrinos and Kaluza-Klein modes on the  $S_1$  are encountered, with dimensions of order  $\sim 10^{30}$ , and then even larger gaps to more and more irrelevant operators corresponding to the electron, muon, pions and the rest of the Standard Model spectrum. All the details of the both the Standard Model and whatever comes beyond it are contained in the spectrum of ridiculously irrelevant operators in the CFT.

Of course CFT's with this type of huge gap in their spectrum of operators have long been known to be relevant to duals of string theory models compactifying to AdS with fixed moduli. Indeed, the peculiarity of the CFT's led some to speculate that such CFT's are impossible and that there had to be some hidden inconsistency in these constructions. Here we see that precisely such CFT's arise even in the simplest possible case of 2D theories as the duals of the  $AdS_3 \times S_1$  vacuum of the Standard Model. Conversely, if it is ever possible to prove that CFT's with these properties do not exist, this necessarily implies that the deep IR spectrum of our world must have additional light states to remove the AdS minimum of the radion potential!

Our  $AdS_3$  minima are certainly metastable; there may be deeper  $AdS_3$  minima in the Standard Model landscape. Whatever the deepest such minimum is, could it absolutely stable? This would be surprising given that the background is completely non-supersymmetric. One possible instability would be the nucleation of a Witten bubble of nothing [7] but this requires antiperiodic fermions around the circle while our vacuum exists only for periodic fermions. A more fundamental issue is that, since the bulk 4D theory has a positive cosmological constant and a  $dS_4$  vacuum, we expect on general grounds that this  $dS_4$  solution should be unstable to tunneling into other parts of the larger landscape. The  $dS_4$  decays via bubble nucleation; the bubble size  $R_4$  can range in size from micro-physical scales to as large as the  $dS_4$  Hubble length  $\ell_4$ , the latter arising from the minimal possibility of Hawking-Moss transitions out of de Sitter space on Poincare recurrence times. If our cosmological constant is tuned to be small by the presence of a huge discretum of nearby vacua,  $R_4$  is parametrically smaller than  $\ell_4$ , it is conceivable that  $R_4 \sim \ell_4$  if our vacuum is isolated by huge potential barriers from the rest of the landscape.

How is the apparently necessary  $dS_4$  instability reflected in the  $CFT_2$  dual of the  $AdS_3$  vacuum? If  $R_4$  is smaller than the size of the  $S_1$ ,  $\sim 20$  microns, it is clear that the same bubble nucleation process will occur in the  $AdS_3 \times S_1$  vacuum. Actually, even if  $R_4$  is only smaller than the  $AdS_3$  length, there is an effective 3D bubble that mediates the decay: if the domain wall bounding the surface of the 4D bubble has surface tension  $\sigma$  and the energy difference between vacua is  $p$ , we have  $R_4 \sim \sigma/p$ ; wrapping the wall on the circle gives us a lower-dimensional wall of tension  $\sigma r$  while the pressure difference is  $pr$ , so a 3D bubble has a size  $R_3 \sim (\sigma r)/(pr) \sim R_4$ . For our neutrino-supported  $AdS_3$  vacua, the  $AdS_3$  length is only few times smaller than the  $dS_4$  Hubble; so if there is a discretum of vacua



**Figure 4:** a) Extremal Reissner-Nordstrom black hole interpolates between asymptotically flat region and  $\text{AdS}\times\text{S}$  vacuum; b) Extremal black hole interpolating to the lower dimensional vacuum stabilized by the Casimir effect.

allowing for the adjustment of our cosmological constant, the  $\text{AdS}_3$  must also be unstable, with an exponentially long lifetime.

Presumably this means that the CFT must itself be ill-defined at a tiny non-perturbative level;<sup>1</sup> for instance by having a marginal perturbation  $g$  with a metastable minimum and an unbounded below potential. The timescale of the instability of the CFT could be of order  $\ell_3 e^{-1/g}$ . If our vacuum is isolated by huge barriers from the rest of the landscape, it is conceivable that the AdS vacua are absolutely stable, since the required bubble, while being smaller than  $\ell_4$ , could be larger than  $\ell_3 \sim \ell_4/4$ , though this seems incredibly unlikely!

#### 4. Quantum horizons

Given the existence of a landscape of vacua in the Standard Model, it is natural to ask whether it is possible to find geometries interpolating between vacua with a different number of non-compact space dimensions. Such interpolations are already familiar for classical  $\text{AdS}_n\times\text{S}_m$  vacua. For instance, the Standard Model possesses  $\text{AdS}_2\times\text{S}_2$  vacua with the sphere stabilized by a flux of the electric field. The interpolating solution is nothing but the extremal Reissner-Nordstrom black hole. Indeed, far from the black hole the metric is flat, while in the vicinity of the horizon an infinite  $\text{AdS}_2\times\text{S}_2$  throat is developed, see figure 4a.

It has been fruitful to view extremal black holes as interpolations between different vacua (cf. [8]) in the context of the interpolation of the scalar moduli fields in supersymmetric theories between spatial infinity and the black hole horizon (“attractor mechanism”).

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<sup>1</sup>We thank Juan Maldacena and Nathan Seiberg for a discussion on this point.

As we will show, this viewpoint is useful in broader context. In particular, for the Casimir stabilized vacua this leads to black hole solutions with the horizon supported entirely by the quantum effects (the Casimir energy).

There is a qualitative difference between classical  $\text{AdS}_2 \times S_2$  and the Casimir vacua. In the former case the radius of the sphere  $S_2$  is of the same order as the  $\text{AdS}_2$  length, while for the Casimir vacuum we have a true compactification, where the size of the compact space is much smaller than the curvature length along the non-compact coordinates. Related to this, in the Casimir compactification the radion mass  $m_R$  is much lighter than the compactification scale  $1/R$ . The length scale at which the interpolation happens is determined by the inverse mass of the radion  $1/m_R$ . This is of order  $R$  for the classical  $\text{AdS}_2 \times S_2$  vacuum so the interpolating geometry has a form of a “hole”. Instead, for the Casimir vacuum the interpolating geometry takes the form of a cone with a narrow opening angle (see figure 4b)).

### 4.1 Setting up the problem in the three-dimensional case

Let us start a more explicit analysis by exploring geometries interpolating between three-dimensional and two-dimensional vacua. As we will see this case turns out to be remarkably simple technically but contains much of non-trivial physics. It is natural to look for an interpolating metric with the following form

$$ds^2 = -A^2(z)dt^2 + dz^2 + R^2(z)d\phi^2, \tag{4.1}$$

where  $\phi \in [0, 2\pi)$  is a periodic coordinate. The precise form of the energy-momentum tensor is determined by the specific mechanism used to stabilize the two-dimensional vacuum. It is straightforward to calculate the energy-momentum related to the classical contributions to the radion potential. For instance, if the cosmological constant in three dimensions is negative, one can obtain a stabilized lower dimensional  $\text{AdS}_2 \times S_1$  vacuum by turning on a flux of a scalar axion field  $\Phi$  (note, that in three dimensions this is equivalent to having the electromagnetic flux),

$$\Phi = F\phi.$$

Then, by explicit computation, the axion energy-momentum tensor in the geometry (4.1) is

$$T_M^N = - \begin{pmatrix} \rho(R)\delta_\mu^\nu & 0 \\ 0 & \sigma(R) \end{pmatrix}, \tag{4.2}$$

where

$$\rho(R) = \frac{F^2}{R^2}$$

is the classical contribution to the radion potential coming from the gradient energy of the axion field, and

$$\sigma(R) = \rho(R) + R \partial_R \rho(R). \tag{4.3}$$

Actually, the relation (4.3) between  $T_\phi^\phi$  and  $T_\mu^\nu$  is a direct consequence of the conservation of the energy-momentum tensor of the form (4.2) in the metric (4.1).

This was a classical example. The Casimir contribution to the energy-momentum for the geometry (4.1) is in principle more involved. Indeed, the compactification scale  $R$  is changing in space, so the one-loop contribution is a complicated functional depending on the local value of  $R(z)$  as well as on all its derivatives. Fortunately, we do not need the exact form of this functional for our purpose of finding the interpolating solutions. Indeed, as we argued before, we expect  $R(z)$  to be a very slow varying function of  $z$ , so that locally the geometry is well approximated by a cylinder, and the derivative part of the Casimir energy can be safely ignored. Under this assumption, because of the Lorentz invariance, the energy-momentum tensor is again of the form (4.2) where  $\rho(R)$  is determined by the Casimir energy,

$$\rho(R) = \frac{V_C(R)}{2\pi R} .$$

We proceed with general  $\rho(R)$ , and will be more specific about its shape later, when necessary. Of course, as shown in the appendix A, a  $T_M^N$  of the form (4.2) agrees with the explicit calculation of the Casimir energy-momentum.

To summarize, we need to study solutions of the three-dimensional Einstein equations for the metric ansatz (4.1) with the energy-momentum of the form (4.2). Explicitly, these equations are

$$M_3 R'' = -R \rho(R) , \tag{4.4}$$

$$M_3 A' R' = -A R \rho(R) , \tag{4.5}$$

$$M_3 A'' = -A [\rho(R) + R \partial_R \rho(R)] , \tag{4.6}$$

where  $M_3$  is the three-dimensional Planck mass. For the two-dimensional vacua the radius of the compact dimension is constant  $R = R_0$  so they correspond to zeros of the Casimir energy,

$$\rho(R_0) = 0 ,$$

while the curvature along the non-compact dimensions is determined by the slope of  $\rho$ ,

$$A(z) = \begin{cases} \exp(z/\ell_2), & \text{AdS}_2 \times \text{S}_1 \text{ vacuum } (\rho' < 0) \\ 1, & \text{M}_2 \times \text{S}_1 \text{ vacuum } (\rho' = 0) \\ \cos(z/\ell_2), & \text{dS}_2 \times \text{S}_1 \text{ vacuum } (\rho' > 0) \end{cases} . \tag{4.7}$$

These coordinates cover the Poincare and causal patches of  $\text{AdS}_2$  and  $\text{dS}_2$ , but can be clearly extended to the global  $\text{AdS}_2$  ( $\text{dS}_2$ ).

For solutions with non-constant  $R(z)$  one can take the ratio of the  $(tt)$  and  $(zz)$  equations (4.4) and (4.5), and arrive at the following relation between  $A$  and  $R$ ,

$$A(z) = R'(z) . \tag{4.8}$$

As a result the interpolating metric (4.1) takes the form

$$ds^2 = -R'^2 dt^2 + dz^2 + R^2 d\phi^2 \tag{4.9}$$

and the  $(tt)$  equation (4.4) implies that  $R$  is a solution to the one-dimensional mechanical problem with the effective potential  $U$  determined by

$$\frac{dU}{dR} = M_3^{-1} R \rho(R). \quad (4.10)$$

Note that this potential is extremal at the values of  $R$  corresponding to the lower dimensional vacua. From eq. (4.9) we see a direct confirmation of the intuition that the interpolation to the lower dimensional vacuum takes place in the near horizon limit — the region where the radius of the compact dimension approaches a constant value,  $R' \rightarrow 0$ , corresponds to the horizon of the metric (4.9). To understand better the causal structure of the metric (4.9) it is convenient to perform a change of coordinates and use  $R$  itself as the interpolating variable. With this choice of coordinates the metric (4.9) is

$$ds^2 = -f(R)dt^2 + f(R)^{-1}dR^2 + R^2d\phi^2, \quad (4.11)$$

where  $f(R) = R'^2$  can be found explicitly by making use of the “energy” conservation law of the mechanical problem (4.4), giving

$$f(R) \equiv R'^2 = \epsilon - U(R). \quad (4.12)$$

The  $(tR)$  part of the metric (4.11) has the typical form of black hole geometries, with horizons located where  $f(R)$  is zero. We see that the metric ansatz (4.1), having the advantage of making the interpolating nature of the solution explicit, actually covers only a small part of the interpolating geometry.

For metric written in the form (4.1) we found two branches of solutions — lower dimensional vacua (4.7) and solutions with non-constant  $R(z)$ , described by the mechanical problem (4.10). The latter can be presented in the form (4.11). To recover the compactified vacuum solutions with (4.11) let us choose  $\epsilon = U(R_0)$  and zoom on the part of the geometry (4.11) where  $R$  is close to  $R_0$ . Namely, let us write

$$R = R_0(1 + \alpha r)$$

and rescale  $t \rightarrow \tau = \alpha R_0 t$ . Taking the limit  $\alpha \rightarrow 0$  we obtain the  $\text{AdS}_2 \times \text{S}_1$  ( $\text{dS}_2 \times \text{S}_1$ ) metric for negative (positive)  $U''(R_0)$  in the form (4.11),

$$ds^2 = -\frac{U''(R_0)}{2} r^2 d\tau^2 + \frac{2}{U''(R_0)} \frac{dr^2}{r^2} + R_0^2 d\phi^2,$$

with curvature length

$$\ell_2 = \sqrt{\frac{2}{|U''(R_0)|}}. \quad (4.13)$$

Finally, let us recall that our solutions are trustworthy as long as the radius  $R$  changes slowly along the non-compact coordinates. When the metric is written in the form (4.9) this implies  $R' \ll 1$ . From the definition (4.12) we see that this condition is translated in the frame (4.11) to

$$f(R) \ll 1.$$

Given that this condition is satisfied one can trust the metric (4.11) both in the regions where  $f(R)$  is positive and negative, i.e. irrespectively of whether the size of the compact dimension changes in space or time.

## 4.2 Extremal black holes interpolating from $M_3$ to $AdS_2 \times S_1$

To be concrete, let us first focus on the case when the three-dimensional cosmological constant is zero and the axion fluxes are absent, so that the effective potential  $U$  goes to zero at large values of  $R$ . Let us start with the simplest case, when the lower dimensional vacuum has a negative cosmological constant. According to (4.7) this implies that the effective potential has a maximum at  $R = R_0$ , see figure 5. A potential of this form is generated, for instance, in a theory with some number of light bosons and heavy fermions, such that the total number of fermionic degrees of freedom is larger than the total number of bosons. In the simple case when all fermion masses are characterized by a single scale  $\mu \ll M_3$  all other parameters in the effective potential  $U$  are also determined by this scale. For instance, the radius of the compact dimension in the two dimensional vacuum is

$$R_0 \sim \mu^{-1},$$

while the value of the potential at the maximum is

$$U(R_0) \sim \frac{\mu}{M_3}.$$

The solution to the one-dimensional mechanical problem, which determines the shape  $R(z)$  of the interpolating geometry, has energy  $\epsilon = U(R_0)$ , so that it reaches the top of the effective potential in an infinite “time”  $z$ . For  $z \rightarrow +\infty$  it describes a flat (2+1) dimensional space with opening angle equal to

$$\theta_o = 2\pi R' \Big|_{z=+\infty} = 2\pi \sqrt{U(R_0)} \sim 2\pi \left( \frac{\mu}{M_3} \right)^{1/2}. \quad (4.14)$$

At  $z = -\infty$  the radius of the compact dimension is exponentially approaching its stabilized value

$$R = R_0(1 + e^{-|z|/\ell_2}),$$

so that the metric (4.9) indeed asymptotes to  $AdS_2 \times S_1$  where the curvature radius  $\ell_2$  is determined by (4.13)

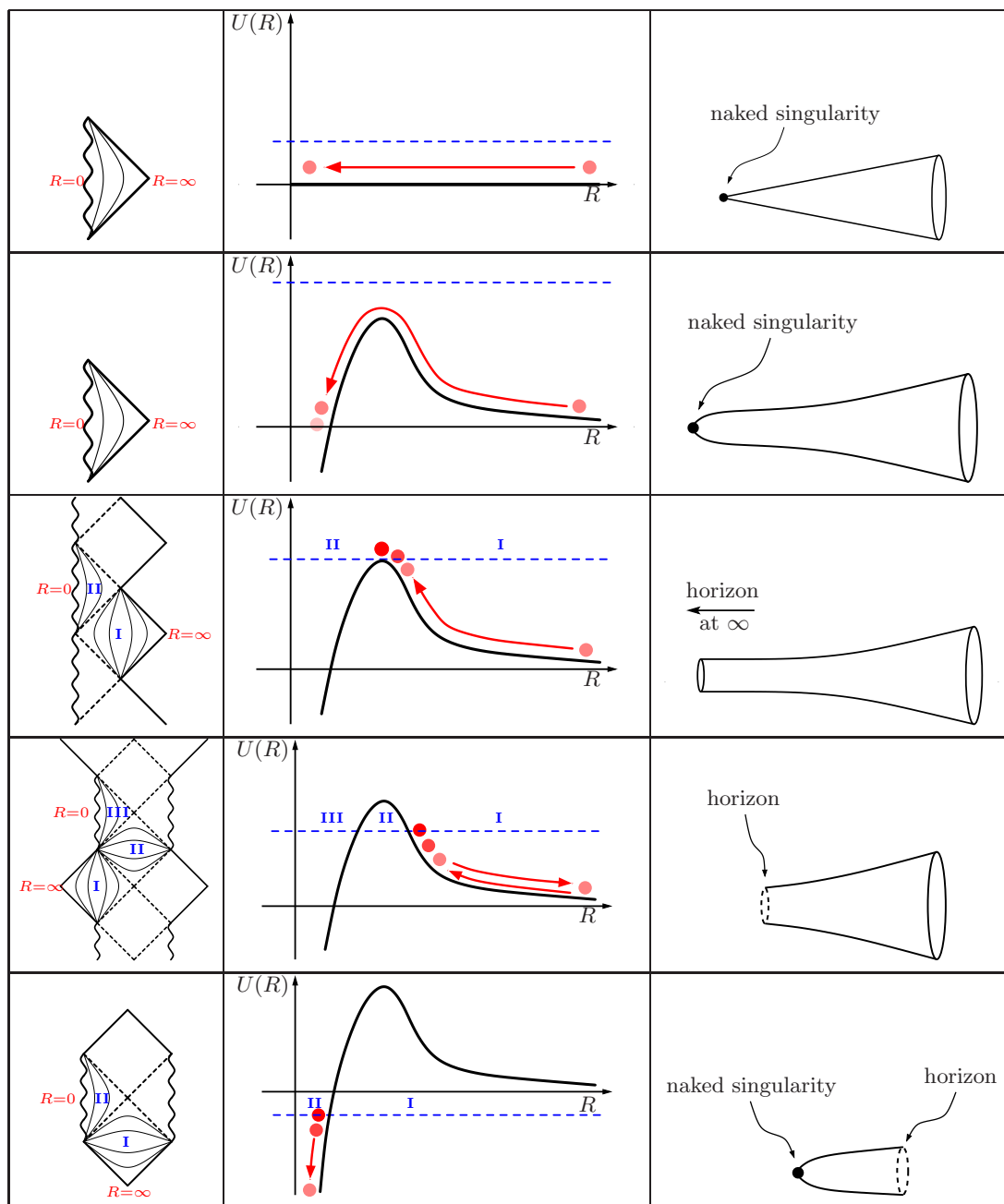
$$\ell_2 \sim \mu \left( \frac{M_3}{\mu} \right)^{1/2}.$$

So, as expected, the interpolating geometry has the form of a narrow cone with a conical singularity resolved into an infinitely long tube. The circumference of the horizon of the interpolating black hole is

$$L = 2\pi R_0 \sim 2\pi \mu^{-1}. \quad (4.15)$$

The surface gravity at the horizon, which is proportional to the first derivative of the function  $f(R)$ , vanishes, so that the Hawking temperature is zero. So, we indeed obtained the asymptotically flat extremal black hole solution in three dimensions. These are not possible in classical gravity, but accounting for the Casimir effect leads to the appearance of the quantum horizon. It is worth stressing again, that the existence and the shape of these quantum black holes is under full control in the limit when the opening angle is small,





**Figure 5:** Interpolating solutions from  $M_3$  to  $AdS_2$  for trivial (1st row) and non-trivial Casimir (2nd÷5th row) mechanical potential (drawn in the 2nd column)  $U(R)$ , defined in eq. (4.10) for different choices of the total energy (blue dashed line). In the first column there are the corresponding Penrose diagrams for the extended solution (eq. (4.11)), with iso- $R$  curves explicit. In the third column the actual geometry is shown for the classical allowed regions of the potential (corresponding to the red arrow curve).

which is true whenever the fermion mass scale is parametrically smaller than the Planck mass.

Interestingly, the Bekenstein entropy for these solutions is determined just by the classical geometry (opening angle) and does not depend on the Planck mass,

$$S = \frac{L}{4G_3} \propto \frac{1}{\theta_o^2}, \tag{4.16}$$

where  $G_3 = 1/(8\pi M_3)$ . In particular, the entropy remains finite in the decoupling limit, when one sends  $M_3$  to infinity while keeping an opening angle fixed. This is understandable, because the mere existence of the three dimensional black holes is due to the quantum effects, so their number of microstates should remain finite in the limit  $\hbar \rightarrow 0$ . In this limit the quantum horizon shrinks to zero, so that one is left with a non-gravitational theory on a cone. Interestingly, this is similar to what happens to the extremal supersymmetric black holes in string theory, where the Bekenstein entropy also remains finite in the limit of zero string coupling. This is one of the crucial ingredients allowing to perform the microscopic calculations of the black hole entropy by counting the BPS D-brane configurations in the decoupling limit [9]. It would be very interesting to understand what are the relevant microscopic degrees of freedom in the decoupling limit for the string realization of our (non-supersymmetric!) setup.

If this is an extremal black hole what charge does it carry? Recall that the low dimensional vacuum only exists with *periodic* boundary conditions for fermions. This is an “exotic” choice. In general, on any simply connected space that asymptotes to a cone, fermions would be antiperiodic in the conical region (for instance, if we replaced the black hole with a smooth “cigar” tip). This antiperiodicity is a reflection of the “minus” sign that the fermionic wave function picks up if one performs a  $2\pi$  rotation around the tip. Choosing the periodic boundary condition on the semi-infinite cylinder corresponds to switching on the  $\mathbb{Z}_2$  flux of the spin connection at the tip, similarly to how the non-integer Aharonov-Bohm flux changes the periodicity of the fermion wave function around a solenoid. This is the flux that labels our interpolating solution.

### 4.3 Non-extremal quantum black holes

The above discussion makes it natural to look for a family of non-extremal quantum black holes carrying  $\mathbb{Z}_2$  flux, such that the interpolating solution is the limiting point for this family with the minimum mass. Also one may wonder whether quantum black holes exist in the sector with trivial flux (anti-periodic conditions for fermions). It is straightforward to identify what are these non-extremal black holes. Let us start with the charged ones and look at the solutions to our mechanical problem with different values of the energy  $\epsilon$ . If  $\epsilon > U(R_0)$ , i.e., the conical opening angle at  $z = +\infty$  is larger than for the extremal solution, a solution in the analogue mechanical problem overshoots  $R = R_0$  and the function  $f(R)$  does not have zeroes. This means that the Casimir energy is not strong enough to shield the tip of the cone by the horizon, and a naked conical singularity develops.

On the other hand, for values of  $\epsilon$  smaller than the energy at the top of the effective potential  $U$ , the solution to the analogue problem undershoots  $R_0$ . As a result  $f(R)$  is zero at the turning point  $R_h > R_0$ , implying that the conical singularity is shielded by a horizon. There is also an inner horizon corresponding to the second zero of  $f(R)$ , so the

causal structure of the extended solution is similar to that of the conventional Reissner-Nordstrom black hole.

Unlike the extremal ones these black holes have non-zero Hawking temperature. It is most easily found by performing the Wick rotation and identifying the periodicity of the Euclidean time. As usual, one obtains that the Hawking temperature is determined by the surface gravity, or explicitly,

$$T_H = \frac{f'(R_h)}{4\pi\epsilon^{1/2}}. \quad (4.17)$$

It is straightforward to check that the Bekenstein entropy in eq. (4.16) satisfies the first law of thermodynamics

$$dM = TdS, \quad (4.18)$$

where the mass  $M$  is determined by the opening angle

$$M = 2\pi M_3 \left(1 - \frac{\theta_o}{2\pi}\right) = 2\pi M_3(1 - \epsilon^{1/2}).$$

Indeed, by definition  $f(R_h) = 0$ , so, taking into account (4.12), one obtains

$$d\epsilon = -f'(R_h)dR_h. \quad (4.19)$$

Using (4.19) one immediately finds that the first law of thermodynamics (4.18) indeed holds.

The non-extremal solutions take an especially simple form in the limit when the opening angle is so small that the radius of the compact dimension at the horizon is much larger than the mass scale of all massive particles. In this limit the Casimir energy is just

$$\rho(R) = -\frac{\zeta(3)n_0}{(2\pi)^4 R^3},$$

where  $n_0$  is the total number of the massless degrees of freedom. Plugging this Casimir potential into (4.10) and (4.12) and performing the rescalings  $t \rightarrow \epsilon^{1/2}t$  and  $R \rightarrow R\epsilon^{-1/2}$  one recognizes in the  $(tR)$  part of the metric (4.11) the radial part of the (3+1)-dimensional Schwarzschild metric with Schwarzschild radius

$$r_s = \frac{\zeta(3)n_0}{M_3 2\pi\theta_o^3},$$

where the asymptotic opening angle  $\theta_o$  of the compact  $\phi$ -coordinate is related to the “energy”  $\epsilon$  in the same way as before,  $\theta_o = 2\pi\epsilon^{1/2}$ . Unlike the charged extremal black hole these solutions do not have a smooth decoupling limit. Indeed, in the limit of large  $M_3$  with fixed opening angle  $\theta_o$  (so that the Bekenstein entropy remains finite), the Hawking temperature  $T_H = (4\pi r_s)^{-1}$  diverges and one cannot trust the semiclassical geometry.

Actually, such non-extremal quantum black holes were known before [10, 1], and were constructed in a way that provides a complementary viewpoint to understand their origin, and simultaneously serves as a nice consistency check for our calculation. Namely, the metric (4.11) with function  $f(R)$  of the Schwarzschild form was found to describe black holes

localized on the Planck brane in the AdS<sub>4</sub> Randall-Sundrum setup. From the holographic CFT point of view these are black holes in three-dimensional gravity coupled to the large  $N$  CFT. Now, at the classical level there is no attractive force in three-dimensional gravity, and the only effect of the point mass on the geometry is to produce the conical deficit angle, so there can be no horizon. This is no longer true at the quantum level; the one loop correction to the graviton propagator gives rise to an attractive potential [11] and as a result the existence of a horizon becomes possible. On the AdS side these quantum effects are captured by the classical dynamics in the bulk, so that the induced metric on the Planck brane indeed describes the quantum black hole geometry in the lower dimensional theory. Of course, the attractive one-loop potential is generated for a general matter sector as well, not just for the large  $N$  CFT, and “Schwarzschild” solutions can be found in this way in the purely three-dimensional setup as well (see, e.g. [12]).

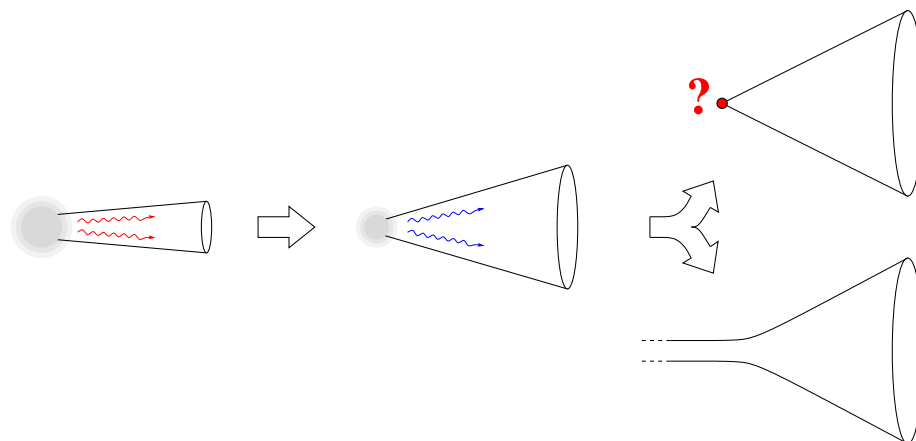
There is a little puzzle here — the one-loop correction to the graviton propagator leads to the attraction, independently of whether a particle circling around the loop is boson or fermion. On the other hand, for the existence of the compactified vacuum and of the extremal interpolating solution is crucial that fermions contribute to the Casimir energy with the opposite sign. The resolution is related to the  $\mathbb{Z}_2$  flux discussed above. In the absence of the flux, the fermions are antiperiodic and the one-loop potential is necessarily attractive. Turning on the flux leads to periodic boundary conditions, making their contribution to the one-loop potential repulsive.

Finally, there are also solutions with negative energy  $\epsilon$ . The meaning of these geometries is not apparent with the metric ansatz (4.9), as the only solutions of the mechanical problem that reach the  $R = \infty$  region in this case are those with the imaginary “time”  $z$ . However, presenting the metric in the form (4.11) makes it explicit that these are as meaningful solutions of the Einstein equations as those with positive energy  $\epsilon$ . Unlike the latter, solutions with negative energy do not asymptote to the conical geometry in the asymptotically flat (large  $R$ ) region. Instead, they describe anisotropic cosmologies with  $R$  playing the role of time. In the large  $R$  region they take the form

$$ds^2 = -\frac{dR^2}{|\epsilon|} + |\epsilon|dt^2 + R^2d\phi^2 .$$

Locally this is just a Minkowski metric, with the  $(R\phi)$  part of it being the expanding Milne universe. Globally there is a difference from the Milne universe due to the compactness of the  $\phi$ -coordinate.

In figure 5 we collected together the different options discussed above — large energies corresponding to the naked conical singularities, critical energy  $U(R_0)$  (extremal black hole), small positive energies (non-extremal black holes) and negative energies (cosmologies). We also presented a schematic cartoon of the geometry in each case, and the corresponding Penrose diagrams. In particular, we see that the conformal diagram corresponding to solutions with negative energy has the same form of the Penrose diagram for Schwarzschild black holes rotated by ninety degrees. This diagram describes the anisotropic bouncing cosmology, where the radius of the compact dimension starts at infinity and bounces back. The scale factor  $f(R)$  in front of the non-compact spatial (in



**Figure 6:** As the quantum black hole evaporates its conical opening angle opens up. Depending on the presence of  $\mathbb{Z}_2$  flux this process either stops at the critical opening angle (4.14) or continues until the horizon shrinks to the Planckian size.

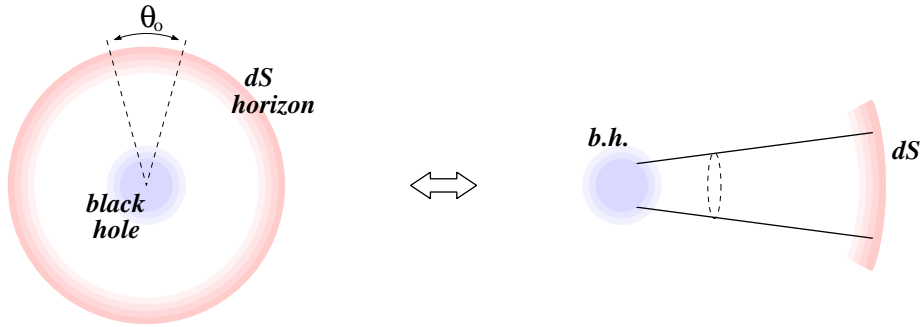
the asymptotically Minkowski region) coordinate  $t$  bounces as well. The big crunch/big bang singularity is partially resolved by the Casimir energy, in a sense that observers can survive a transition from the contracting to the expanding stage without ever hitting a time-like singularity at  $R = 0$ . Note, that similarly to the inner horizon of the Reissner-Nordstrom black hole, the horizon replacing the big crunch singularity suffers an instability with respect to the small perturbations of the initial data.

The quantum black holes that do not carry the  $\mathbb{Z}_2$  flux are also straightforward to identify. In this case fermions satisfy antiperiodic boundary conditions, so that their contribution to the Casimir energy has the same sign as bosons. The solutions of the corresponding mechanical problem describe either bouncing cosmologies with an unresolved singularity (imaginary time solutions with negative energies), or uncharged quantum black holes (positive energy solutions with a single turning point).

This discussion implies the following evolution history for the quantum black holes (see figure 6), after one takes into account the Hawking evaporation. One starts with a black hole of a near critical Planckian mass, which is a very narrow cone with a singularity shielded by the quantum horizon. As a result of the Hawking evaporation the horizon shrinks and the angle of the cone opens up. Depending on whether the  $\mathbb{Z}_2$  flux is present or not, this process either stops at the critical opening angle (4.14) and the extremal black hole forms, or continues until the cone opens completely and the horizon shrinks to the Planckian size.

#### 4.4 Interpolation from AdS<sub>3</sub> and dS<sub>3</sub> vacua

There are no difficulties in extending the above discussion to the case when the three dimensional vacuum is either AdS<sub>3</sub> or dS<sub>3</sub>. All general results of the section 4.1 still apply, the only difference being that the effective potential does not vanish at  $R = \infty$ .



**Figure 7:** Quantum black hole in  $dS_3$  has a narrow opening angle that cuts out most of the causal patch.

For instance, in the  $AdS_3$  case the effective potential behaves as  $U(R) \propto -R^2$ . As before, the extremal interpolating geometry corresponds to the critical solution of the mechanical problem with  $\epsilon = U(R_0)$ . The large  $R$  region of the metric (4.11) asymptotes now to the boundary of the  $AdS_3$ .

Just as in the flat case for larger values of  $\epsilon$  one obtains  $AdS_3$  geometries with a naked conical singularity, and for smaller values of  $\epsilon$  non-extremal black holes. The only difference with the flat case is the absence of the solutions that approach the asymptotically  $AdS$  region as cosmologies.

In a sense, the situation is the opposite in the case of the asymptotic  $dS_3$  geometry. Namely, in this case the potential of the mechanical problem is positive at large  $R$ ,  $U(R) \propto R^2$ , so that at any value of energy  $\epsilon$  the large values of  $R$  belong to the classically forbidden region of the auxiliary mechanical problem. In analogy to what we had at  $\epsilon < 0$  in section 4.3, this implies that  $R$  plays the role of time in this region, so that the metric (4.11) describes an inflationary three-dimensional Universe at the largest values of  $R$ . A turning point of the mechanical solution at large  $R$  corresponds to the horizon of the static patch of  $dS_3$ . As before, at large values of  $\epsilon$  the solution (4.11) does not have any other horizons and develops a naked conical singularity at the origin  $R = 0$  of the static patch. For extremal solutions with  $\epsilon = U(R_0)$  this singularity is resolved into an infinite  $AdS_2 \times S_1$  throat. At even smaller values of  $\epsilon$  it is shielded by a non-extremal horizon. Smallness of  $\epsilon$  implies that these solutions contain only a tiny fraction of the de Sitter horizon, see figure 7.

One peculiarity of the asymptotically  $dS_3$  case, is the existence of a new extremum (minimum) of the effective potential  $U(R)$  at  $R = R_1$ . According to the discussion of section 4.1 this minimum corresponds to the  $dS_2 \times S_1$  vacuum. As a result, solutions (4.11) with  $\epsilon$  close to  $U(R_1)$  develop a number of new interesting features. We will discuss these in section 4.6, where we describe interpolation to the  $dS_2 \times S_1$  vacua.

Here we would like to discuss another important question related to the interpolation from the higher dimensional de Sitter space. Namely, the finite entropy  $S$  of the de Sitter horizon strongly suggests that the Hilbert space describing possible quantum states of the de Sitter space is finite dimensional. In turn, together with the thermal nature of the de Sitter vacuum, this implies that the de Sitter observer should go through all possible states

before the Poincare recurrence time  $\sim e^S$ .

In the context of the metastable de Sitter vacuum, which can decay to the Minkowski or AdS vacuum with the same number of spatial dimensions, this expectation is supported by the remarkable fact that the semiclassical decay rate described either by the Coleman-de Luccia [13] or Hawking-Moss [14] instantons is always faster than  $e^{-S}$ , no matter how high the barrier between the two vacua is. On the other hand, we have not found solutions which have the interpretation of the expanding bubble of the lower dimensional vacuum in the higher dimensional one. How is that compatible with the argument sketched above, that the de Sitter space should be able to populate all other states at the times scales shorter than the recurrence time?

The existence of the interpolating black hole solutions found here indicates that this process is rather different from the conventional Coleman-de Luccia vacuum decay. Namely, instead of creating an expanding bubble of the new vacuum, de Sitter thermal fluctuations may lead to the collapse of most part of the static patch into the quantum black hole described here. Afterwards this black hole will Hawking evaporate and approach an extremal interpolating solution. We did not attempt to find an explicit instanton solution describing such a process. Such an instanton may have rather peculiar properties, as it should change the value of the  $\mathbb{Z}_2$  charge within a causal patch. Note, that a creation of a pair of extremal black holes (such a configuration is neutral with respect to  $\mathbb{Z}_2$ ) is not possible within one causal patch, because each of the black hole has a deficit angle close to  $2\pi$ . On the other hand, there is no conservation law for the charge within a given causal patch and, consequently, no reasons to expect that such an instanton does not exist. Note that unlike for the usual Coleman-de Luccia bubble this transition does not change the microscopic structure of the vacuum, so that small enough observers (for instance, many of the *Amoebosoa*) are able to survive it.

#### 4.5 Interpolation in 4D

Let us now discuss how the interpolating solutions look like in more realistic situations, namely let us describe solutions interpolating from four- to three-dimensional vacua. For simplicity, we will mainly focus on the solutions interpolating from the four-dimensional Minkowski space to  $\text{AdS}_3 \times S_1$ . As we discussed in section 2 this case is relevant for the Standard Model neutrino vacua, in the approximation when one neglects the effects related to the presence of the four-dimensional cosmological constant. In this case we are looking for a cosmic string-like geometry, so that a natural generalization of the three-dimensional ansatz (4.1) is

$$ds^2 = A^2(z) (-dt^2 + dx^2) + dz^2 + R^2(z)d\phi^2, \tag{4.20}$$

where  $x$  is the non-compact spatial coordinate along the string. Note, that *a priori* there is no reason to assume Lorentz invariance in the  $(tx)$  plane, as we did in the ansatz (4.1). As we will see, assuming this symmetry allows to obtain the extremal interpolating geometry, while giving up this symmetry will lead to the related family of non-extremal black objects. For simplicity let us proceed with the Lorentz invariant ansatz (4.1). The energy-momentum tensor still takes the form (4.2), where now, of course,  $\mu, \nu = t, x, \phi$ . The  $(tt)$ ,

$(zz)$  and  $(\phi\phi)$  components of the Einstein equations then take the following form

$$M_4^2 \left( R'' + R' \frac{A'}{A} + R \frac{A''}{A} \right) = -R \rho(R), \quad (4.21)$$

$$M_4^2 \frac{A'}{A} \left( 2R' + R \frac{A'}{A} \right) = -R \rho(R), \quad (4.22)$$

$$M_4^2 \left[ \left( \frac{A'}{A} \right)^2 + 2 \frac{A''}{A} \right] = -[\rho(R) + R \partial_R \rho(R)]. \quad (4.23)$$

To proceed it is convenient to solve for  $A'/A$  from the  $(zz)$ -equation (4.22),

$$\frac{A'}{A} = -\frac{R'}{R} \pm \sqrt{\left( \frac{R'}{R} \right)^2 - \frac{\rho}{M_4^2}}. \quad (4.24)$$

To understand the meaning of the sign ambiguity in (4.24), note that the asymptotically flat boundary conditions at  $z = -\infty$  are

$$\left. \frac{R'}{R} \right|_{z=-\infty} = \frac{1}{z} < 0, \quad \left. A' \right|_{z=-\infty} = 0.$$

These correspond to the “−” sign in (4.24) (recall, that we are assuming zero cosmological constant, so that  $\rho(R) \rightarrow 0$  at large  $R$ ). On the other hand, asymptotically flat boundary conditions at  $z = +\infty$  require  $R'$  to be positive and correspond to the “+” sign in (4.24). The existence of two branches in (4.24) indicates that, just as in the three dimensional case, it is impossible to find a smooth solution of the form (4.20) connecting two asymptotically non-compact flat regions at  $z = \pm\infty$  (such a solution would be a Lorentzian wormhole). In what follows we choose the sign “−” in (4.24) so that the asymptotically flat region is at  $z = -\infty$  (this convention is opposite to the one used before, however it is more convenient for the purposes of the present discussion). Then one can take the combination of the  $(tt)$  and  $(\phi\phi)$  equations (4.21), (4.23) that does not contain  $A''$ , and plug (4.24) there. As a result one arrives at the following equation for the radius of the compact dimension alone,

$$R'' + \gamma R' = -\partial_R U, \quad (4.25)$$

where the effective potential  $U$  is determined by

$$\frac{dU(R)}{dR} = \frac{1}{M_4^2} R(\rho - R \partial_R \rho) \quad (4.26)$$

and the friction parameter  $\gamma$  is

$$\gamma = -2 \left( \frac{R'}{R} + \sqrt{\left( \frac{R'}{R} \right)^2 - \rho} \right). \quad (4.27)$$

The shape of the effective potential  $U$  due to the Casimir energy in a theory with the light spectrum of the Standard Model (and with zero cosmological constant) is the same as the one in figure 5. As before, the maximum at  $R = R_0$  corresponds to the compactified



AdS<sub>3</sub>×S<sub>1</sub> vacuum and we are interested in the solution that starts at  $R = \infty$  and makes it to the top of the potential in a infinite time.

The difference with the three-dimensional case is the presence of the friction term in (4.25). It is straightforward to check that  $\rho < 0$  in the whole region to the right of the maximum,  $R > R_0$ , so that the friction parameter  $\gamma$  is negative there and gives rise to an antifriction. The presence of this antifriction does not prevent us from running the argument proving the existence of the extremal solution. Just like in the three-dimensional case in the limit of a very small opening angle,  $R'(-\infty) \rightarrow 0$ , the solution to the mechanical problem (4.25) undershoots the maximum, while for large opening angles it overshoots, so there is a critical value such that  $R(z)$  monotonically drops down and stops at  $R_0$  in an infinite time. From (4.24) one sees that the warp factor  $A(z)$  also monotonically drops down for this solution without ever changing its sign (recall, that we chose the “−” sign in (4.24)) and at large  $z$  approaches zero as

$$A(z) \sim e^{-z\sqrt{-\rho}},$$

so the extremal solution indeed interpolates to the AdS<sub>3</sub>×S<sub>1</sub> vacuum. On dimensional grounds it is clear that the asymptotic opening angle for the solution interpolating to the neutrino vacuum of the Standard Model is

$$\theta_o = 2\pi|R'(-\infty)| \sim \frac{m_\nu}{M_4}.$$

Similarly to the three-dimensional case, solutions with larger opening angles overshoot and develop a conical singularity. On the other hand, the behavior of the solutions with smaller opening angles is different from the lower dimensional case. Namely, as one can see from (4.24), the turning point  $R' = 0$  does not correspond to a horizon any longer, so the undershooting solutions do not describe the non-extremal black strings. What happens instead is that, due to the presence of the antifriction term in (4.25), the radius of the compact dimension diverges at a finite distance after the turning point, so that the solution develops a naked singularity. As we said before, in order to obtain the black non-extremal solutions one has to give up with Lorentz invariance in the ansatz (4.20).

The extension of these results to the AdS<sub>4</sub> case is straightforward. The only subtlety is that the asymptotically AdS<sub>4</sub> boundary condition at  $z = -\infty$  implies that  $A \propto R \propto \exp(|z|/l_3)$ , so that  $R'$  is infinite. This makes it inconvenient to use  $R$  itself as a variable in the auxiliary mechanical problem. Changing variable to  $k = \log R$  in (4.25), one can literally repeat the above argument to prove that the interpolating solution of the form (4.20) exists in this case as well. This solution can be interpreted as a holographic RG flow of a CFT<sub>3</sub> broken by compactifying one of the spatial dimensions on a circle to a CFT<sub>2</sub> in the IR.

However, unlike in the lower-dimensional case, the ansatz (4.20) is not the appropriate one to describe an interpolation from dS<sub>4</sub>. A fast way to see this, is to note that translational invariance in  $x$  is incompatible with dS<sub>4</sub> symmetries. To see this explicitly it is enough to solve eqs. (4.21), (4.22) and (4.23) for a pure cosmological constant,  $\rho(R) = const > 0$ . It is straightforward to check that the resulting vacuum solutions

are never maximally symmetric, i.e.  $dS_4$  metric cannot be presented in the form (4.20). Instead, the cosmic string geometry in the static  $dS_4$  coordinates takes the form

$$ds^2 = -(1 - r^2)dt^2 + \frac{dr^2}{1 - r^2} + r^2 (d\theta^2 + \epsilon \sin^2 \theta d\phi^2) ,$$

where  $\epsilon$  determines the deficit angle. It is likely that the problem of finding an interpolation between this geometry and the  $AdS_3 \times S_1$  vacuum cannot be reduced to ordinary differential equations and requires the analysis of a two-dimensional system of partial differential equations with non-trivial dependence on both  $r$  and  $\theta$ . Having seen how it works in three dimensions, in principle there should be no obstruction for the existence of the quantum black strings in  $dS_4$ .

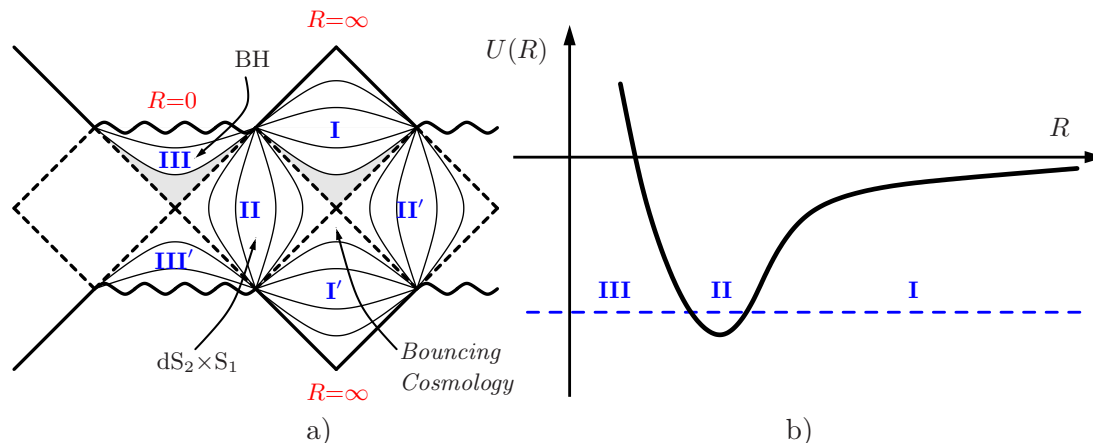
#### 4.6 Interpolation to dS vacua

So far we focused on interpolations to low dimensional vacua with a negative cosmological constant. This situation is similar to the ordinary Reissner-Nordstrom black holes and is relevant for the neutrino vacua of the Standard Model (assuming neutrinos are Majorana). However it is interesting to consider also what happens when the lower dimensional vacuum has a positive cosmological constant.

Natural interpolating solutions in this case are Coleman-de Luccia bubbles describing decompactifications of the lower dimensional vacua as discussed in [15]. Of course, our four-dimensional Universe could not have originated from one of the Standard Model three-dimensional vacua in this way, as the reheating temperature would be too low. However, it would be interesting to study the observational cosmological consequences of the scenario where our Universe was created as a result of the decompactification of a lower dimensional metastable vacuum. We will not address this issue here.

Instead, given that in the three-dimensional setup we have an explicit solution (4.11) that applies to the low dimensional de Sitter vacuum as well, let us discuss its properties in this case. Note that compactifications to two dimensions are somewhat subtle because the radion field is not dynamical. Nevertheless, as discussed in appendix B.3, there is a sense in which the de Sitter vacuum always corresponds to the maximum of the radion potential in this case. Due to the absence of the dynamical radion this vacuum is classically stable under local perturbations (actually, even in four dimensions a de Sitter maximum can be effectively stable if the radion is light enough, so that the Universe is eternally inflating on “the top of the hill”). As a result, instead of the Coleman-de Luccia type of bubbles one may expect the interpolating solution to describe just a classical rolling from the top of the potential in this case.

The shape of the effective potential  $U$  corresponding to the  $dS_2 \times S_1$  vacuum is shown in figure 8b. It is straightforward to analyze the structure of the interpolating solutions (4.11) at different values of  $\epsilon$ . In all cases the corresponding Penrose diagrams are ninety degrees rotations of those shown in figure 5. Let us discuss here the solution exhibiting the richest pattern of features, namely the near extremal one, with  $\epsilon$  being slightly larger than the value of the potential  $U$  at the minimum. The corresponding Penrose diagram is shown in figure 8a.



**Figure 8:** a) Conformal diagram of the near extremal solution interpolating to the  $dS_2 \times S_1$  vacuum and b) the corresponding mechanical potential  $U(R)$ .

Part **II** (as well as its horizontally translated cousins **II'** ...) of this diagram corresponds to the classically allowed region of the mechanical problem. According to the discussion at the end of section 4.1, in the near extremal limit, the geometry of this region is that of the causal diamond of the  $dS_2 \times S_1$  vacuum. As usual, after continuation through the horizon the  $R$ -variable becomes time-like, while the  $t$ -variable is space-like. So parts **I** and **III** of the Penrose diagram cover regions with anisotropic cosmological expansion.

The geometry of the region **III** has a structure somewhat similar to that of the interior of the Schwarzschild black hole. Namely, the compact coordinate shrinks in this region down to zero size at the  $R = 0$  singularity. So, as a result of the quantum effects the conical singularity is replaced by a big crunch singularity for the compact dimension. However, the function  $f(R)$  grows indefinitely in this region, implying that the non-compact space-like coordinate  $t$  experiences superaccelerated cosmological expansion and eventually hits the big rip singularity at  $R = 0$ .

One interesting difference with the black hole interior is due to the part of the region **III** adjacent to the region **II** (grey shaded region in figure 8a), where the effective potential is still approximately quadratic,

$$U \approx U(R_0) + \ell_2^2 (R - R_0)^2 + \dots$$

Plugging this expression into  $f(R)$ , one finds that this part of the region **III** is an exponentially inflating two-dimensional Universe in the FRW coordinates. The size of the compact dimension slowly rolls down here, so in a sense the radion plays the role of the inflaton. This interpretation is somewhat subtle though, because, at least at the classical level, there is no dynamical radion in the compactifications from three to two dimensions.

Finally, it is the existence of the region **I** which signals that we are dealing with an interpolating geometry. Indeed, in this region, the coordinate  $R$  is also time-like and, as it grows to infinity, the function  $f(R)$  approaches a constant value  $f(+\infty) \approx -|U(R_0)|$ , so

that the metric is flat with the  $(R\phi)$  part being the expanding Milne universe. Globally there is a difference from the Milne universe due to the compactness of the  $\phi$ -coordinate.

Just like in the region **III**, a shaded part of the region **I** describes an exponentially inflating two-dimensional Universe. Finally, regions **I'**, **III'** describe the same cosmological solutions as **I**, **III** with the reversed direction of time. In the vicinity of the boundary with the region **II** regions **I'**, **III'** describe collapsing cosmologies and their horizons are very much similar to the inner Cauchy horizon of the Reissner-Nordstrom black hole. As usual, such a horizon is unstable with respect to small perturbations of the initial data, so regions **I'**, **III'** are not to be there in a physically realizable situation.

Consequently, as expected, the physical meaning of the  $dS_2 \times S_1$  interpolating solution is to describe the inflation “on the top of the hill” in the lower dimensional vacuum, which ends up either in the singularity, where the compact dimension collapses, or exits into the asymptotically flat decompactified space-time. It will be interesting to calculate the spectrum of cosmological perturbation for this inflation. As we already mentioned, a peculiar feature of this case is that there are no propagating perturbations of the inflaton (radion) at the classical level. However, at the one-loop level we expect the inflaton to become dynamical.

## 5. Conclusions

We have seen that the Standard Model has a near-moduli space of lower-dimensional vacua with moduli stabilized by a combination of a tiny tree-level contribution from the cosmological constant and one-loop corrections. For the minimal theory of neutrino masses, there are  $AdS_3 \times S_1$  vacua, implying the existence of a dual  $CFT_2$  describing the Standard Model coupled to Quantum Gravity. We also showed quite generally that it is possible to interpolate to lower-dimensional AdS vacua as near-horizon regions of new kinds of quantum extremal black objects — black strings in going from  $4 \rightarrow 3$  dimensions, black holes from  $3 \rightarrow 2$  dimensions. The extremal 3D black holes are particularly interesting — they are metastable objects with an entropy that is independent of  $\hbar$  or  $G_N$ , so a non-gravitational microscopic accounting of their entropy might be possible in a decoupling limit where  $G_N, \hbar \rightarrow 0$  and the geometry degenerates to a cone with a fixed, small opening angle.

There are a number of obvious issues that require further elaboration. We did not study the radion effective potential for radii smaller than the QCD scale, so we don't know if there are additional vacua there. Nor have we analyzed the SM potential in the case of even lower dimensional compactifications. It would be interesting to explicitly find the gravitational solutions that interpolate between  $dS_4$  and 3D vacua — symmetry considerations suggest that the problem is different from its lower-dimensional analogue. We also did not attempt to find interpolating solutions from 4D to 2D vacua. If 3D de Sitter vacua can exist, it is natural to ask if our universe could have originated from tunneling out of eternal inflation in 3D. Of course we need to have a phase of slow-roll inflation *after* the nucleation of our 4D bubble takes place, so the tunneling should happen with the inflaton stuck at the top of its potential. It would be interesting to investigate possible cosmological signatures of such

a scenario. Finally, we have not explicitly constructed instantons for transitions between de Sitter space and the extremal quantum black objects.

A crucial ingredient for both the existence of these non-SUSY vacua and the quantum horizons allowing interpolation is the violation of the null-energy condition and negative gravitational energy associated with the Casimir effect. It is interesting that objects with negative gravitational energy play a crucial role in *all* modern mechanisms for stabilizing moduli to flat or dS spaces such as KKLT [16]; for instance negative tension orientifold planes are present in these constructions. Just as the new Standard Model vacua we have found are associated with quantum black objects, it is natural to conjecture that at least the AdS vacua in the string landscape can be realized as near-horizon geometries of new black brane solutions asymptoting to 10 or 11 dimensions, or more generally some point on the maximally supersymmetric moduli space. The orientifolds must play a crucial role in allowing the existence of these solutions. The landscape of lower-dimensional vacua should thus be associated with a zoo of exotic black hole solutions, allowing us to look at the vacua from the “outside”. It would be interesting to try and find these black brane solutions explicitly for the classical IIA vacua of [17]. As a simpler warm-up with the same essential features — negative tension and fluxes— consider stabilizing a 1D interval (or  $S_1/\mathbb{Z}_2$  orbifold), by having a negative tension  $T$  on one end of the interval and an axion with decay constant  $f$  and fixed periodicity around the circle. Such a situation could well exist for our vacuum; if there is low-energy SUSY, we could have  $T \sim -m_{\text{SUSY}}^4$  and the QCD axion suffices. The radion effective potential is  $V_{\text{eff}}(R) \sim R^{-3}(\Lambda R - |T| + f^2/R)$ ;  $\Lambda$  is negligible here and there is a non-trivial AdS minimum. The interpolating geometry in this case should look like a narrow strip, bounded by the negative tension brane on one end and the other end of the interval on the other, again with a small opening angle.

The necessity of negative energy objects in realistic models of modulus stabilization has sometimes been thought of as a technicality — but we have seen that they are associated with new sorts of horizons and thus surprising causal structures in the higher-dimensional geometries the lower-dimensional vacua are embedded in. It is worth exploring this issue further. For instance, we often imagine tunneling out of stabilized dS vacua to 10/11 dimensional supersymmetric space-times; but this is not correct. The asymptotic spaces must not only carry a remnant of e.g. the fluxes labeling the vacua, but they also have e.g. orientifold planes with negative gravitational energy. How do these affect the geometry?

## Acknowledgments

We thank Tom Banks, Raphael Bousso, Michael Dine, Gia Dvali, Juan Maldacena, Nathan Seiberg, Marco Serone, Steve Shenker, Andy Strominger, Raman Sundrum, Cumrun Vafa and Edward Witten for stimulating discussions. Our work is supported by the DOE under contract DE-FG02-91ER40654.

## A. Casimir energy

In this appendix we review the derivation of the 1-loop Casimir contribution to the energy-

momentum tensor for a generic massive field, in  $d$ -dimensions with one dimension compactified on a circle. Let us call  $x^d = y \in [0, 2\pi R)$  the compact dimension. Given a free scalar field with Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}m^2\Phi^2,$$

at 1-loop the expectation value of the energy-momentum tensor reads

$$\begin{aligned} \langle T_{\mu\nu} \rangle &= \left\langle \mathcal{L} g_{\mu\nu} - 2 \frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} \right\rangle \\ &= \lim_{x' \rightarrow x} \left[ \frac{1}{2} (\partial_\mu \partial'_\nu + \partial_\nu \partial'_\mu) - \frac{1}{2} g_{\mu\nu} (\partial^\rho \partial'_\rho + m^2) \right] G(x - x'), \end{aligned} \quad (\text{A.1})$$

where  $G(x - x') = \langle \Phi(x)\Phi(x') \rangle$  is the free propagator. When one dimension is compact the Casimir contribution can easily be obtained just by summing the infinite volume Green function over all the images, namely

$$G(x - x') = \sum'_n G_\infty(x - x' + 2\pi R n \hat{y}),$$

in the sum  $n$  runs over all integers but 0, which corresponds to the infinite volume  $R$ -independent contribution that must be reabsorbed into the cosmological constant. Notice that, having subtracted the  $n = 0$  contribution, also the second term in eq. (A.1) vanishes. So we finally have

$$\begin{aligned} \langle T_{\mu\nu} \rangle &= \frac{1}{2} \lim_{x' \rightarrow x} (\partial_\mu \partial'_\nu + \partial_\nu \partial'_\mu) \sum'_n G_\infty(x - x' + 2\pi R n \hat{y}) \\ &= - \sum'_n \partial_\mu \partial_\nu G_\infty(y_n) \Big|_{y_n = 2\pi R n \hat{y}} \\ &= - [\rho(R) \eta_{\mu\nu} + R \rho'(R) \delta_\mu^y \delta_\nu^y], \end{aligned} \quad (\text{A.2})$$

where

$$\rho(R) = 2 \sum'_n \frac{\partial G_\infty(y_n^2)}{\partial y_n^2} \Big|_{y_n = 2\pi R n \hat{y}},$$

is the Casimir energy density. In the case of charged fields we can also have non-periodic boundary conditions

$$\Phi(x, y + 2\pi R) = e^{i\theta} \Phi(x, y),$$

and the Green functions in the sum get an extra Wilson line contribution

$$\sum'_n e^{in\theta} G_\infty(x - x' + 2\pi R n \hat{y}).$$

So the final expression for the Casimir energy density in the general case reads

$$\rho(R) = 2 \sum'_n e^{in\theta} \frac{\partial G_\infty(y_n^2)}{\partial y_n^2} \Big|_{y_n = 2\pi R n \hat{y}}. \quad (\text{A.3})$$

This formula applies also for fermion, vector and graviton fields, with an extra minus in the case of fermions. By plugging in the explicit formula for the Green function one can

easily read the result. For example, in the case of  $d = 4$  for a massless field with periodic boundary conditions, the Green function reads

$$G_\infty(y_n^2) = \frac{1}{4\pi^2 y_n^2},$$

so that eq. (A.3) gives

$$\rho(R) = -\frac{4}{(2\pi)^6 R^4} \sum_{n=1}^{\infty} \frac{1}{n^4} = -\frac{\pi^2}{90} \frac{1}{(2\pi R)^4}.$$

The contribution in the effective potential in the dimensionally reduced 3D theory reads

$$V_C = 2\pi R \rho(R) = -\frac{1}{720\pi R^3},$$

while the contribution in the Weyl-rescaled metric of eq. (2.1) is just

$$-\frac{r^3}{720\pi R^6}.$$

From the form of the energy-momentum tensor (A.2) we can easily derive the condition for  $\rho(R)$  not to violate the Null Energy Condition

$$T_{\mu\nu} n^\mu n^\nu \geq 0, \quad \forall n^\mu : n^2 = 0,$$

and reads

$$\begin{aligned} T_{\mu\nu} n^\mu n^\nu &= -2(n^y)^2 R \rho'(R) \geq 0, \\ \rho'(R) &\leq 0, \end{aligned}$$

which is satisfied by fermions but violated by bosons.

Let us now derive the explicit formula for  $\rho(R)$  in the most general case. Since we are interested to the value of the Green function outside the light-cone we can work directly in Euclidean space, the Green function then reads

$$G_\infty(x) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{ikx}}{k^2 + m^2} = \frac{m^{d-2}}{(2\pi)^{d/2}} \frac{K_{d/2-1}(mx)}{(mx)^{d/2-1}}, \quad (\text{A.4})$$

where  $K_\nu(z)$  is the Bessel function

$$K_\nu(z) = \frac{1}{2} \int_0^\infty d\beta \beta^{\nu-1} e^{-\frac{z}{2}(\beta + \frac{1}{\beta})}.$$

Now, by using the fact that

$$\partial_z \left( \frac{K_\nu(z)}{z^\nu} \right) = -\frac{K_{\nu+1}(z)}{z^\nu},$$

inserting the result for the Green function (A.4) into eq. (A.3) we get

$$\rho(R) = -\sum_{n=1}^{\infty} \frac{2m^d}{(2\pi)^{d/2}} \frac{K_{d/2}(2\pi R m n)}{(2\pi R m n)^{d/2}} \cos(n\theta).$$

The massless limit can easily be taken by noticing that for  $z \rightarrow 0$ ,

$$z^\nu K_\nu(z) = 2^{\nu-1} \Gamma(\nu) \left[ 1 - \frac{z^2}{4(\nu-1)} + O(z^4) \right],$$

and reads

$$\rho(R) = -\frac{2}{(2\pi R)^d \Omega_{d-1}} \operatorname{Re} \left[ \operatorname{Li}_d(e^{i\theta}) - \frac{2\pi^2 \operatorname{Li}_{d-2}(e^{i\theta})}{d-2} (mR)^2 + O(mR)^4 \right],$$

where

$$\operatorname{Li}_n(z) \equiv \sum_{k=1}^{\infty} \frac{z^k}{k^n}, \quad \Omega_{d-1} \equiv \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})},$$

$\operatorname{Li}_n(1) = \zeta(n)$ ,  $\operatorname{Li}_n(-1) = (2^{1-d} - 1)\zeta(n)$  and  $\zeta(n)$  is the Riemann zeta-function. Notice also that the first corrections to the massless limit is negative and proportional to  $(mR)^2$ .

Analogously for  $m \rightarrow \infty$ , using

$$z^\nu K_\nu(z) \xrightarrow{z \rightarrow \infty} \sqrt{\frac{\pi}{2}} z^{\nu-\frac{1}{2}} e^{-z},$$

we get

$$\rho(R) \xrightarrow{m \rightarrow \infty} -\frac{(mR)^{\frac{d-1}{2}}}{(2\pi R)^d} e^{-2\pi R m} \cos(\theta), \quad (\text{A.5})$$

which shows the exponential suppression for  $mR > 1$ .

## B. More vacua

### B.1 Other 3D SM vacua

In section 2 we showed how Casimir contributions to the effective potential of the radion may determine a non trivial vacuum, actually a continuum, at the micron scale. One can now ask what happens at shorter distances. For smaller sizes of the radius the neutrinos are effectively massless and since the number of fermionic degrees of freedom is larger than the number of bosonic ones, with periodic boundary conditions the scalar potential grows, independently of the value for the Wilson loop. Nothing new happens until the size of the radius approaches the Compton wavelength of the electron. At this point also the electron d.o.f. start to be important. Moreover, since the electron is charged, also the Wilson loop will start receiving important contributions: for  $\theta = 0$  the contribution to the effective potential is positive and it continues to grow; For  $\theta = \pi$ , on the other hand, the contributions from the fermions is negative, the potential starts decreasing, developing a saddle point at  $R \sim 1/m_e$  and  $\theta = \pi$ . It seems that the structure of the SM potential is getting more and more interesting.

Because in three dimensions the electromagnetic coupling is relevant, one could worry that at large distances the theory becomes strongly coupled and the calculation breaks down, however, it is easy to check that, as long as the 4D coupling is perturbative, this happens only at distances parametrically larger than the radius, and the calculation is always within the regime where it can be trusted.



	$\theta$	$g, \gamma$	$\nu$	$e^-$	$\mu^-$	$\pi$	$K$	$\eta_8$
M	0	-4	2	6	10	7	3	2
M	$\pi$	-4	2	-3/2	-5	-17/4	-9/2	-11/2
D	0	-4	8	12	16	13	9	8
D	$\pi$	-4	8	9/2	1	7/4	3/2	1/2

**Table 1:** Total number of d.o.f. after each threshold weighted with the factors  $\mp 1$  for bosons or fermions and with 1 or  $-7/8$  for charged fields if the Wilson loop value is  $\theta = 0$  or  $\pi$  respectively. The two cases refer to Majorana (M) and Dirac (D) neutrino. A change in sign signals a stationary point.

For smaller radii more and more states come in, changing at each stage the behavior of the potential. If we define the single bosonic contribution to the Casimir energy as

$$V_C^{(1)}[R, m, \theta] \equiv -\frac{r^3 m^4}{\pi R^2} \sum_{n=1}^{\infty} \frac{\cos(n\theta)}{(2\pi R m n)^2} K_2(2\pi R m n),$$

the full effective potential will then read

$$V = \frac{2\pi r^3 \Lambda_4}{R^2} + \sum_a (-1)^{F_a} n_a V_C^{(1)} \left[ R, m_a, 2\pi \left( q_a A_\phi + \frac{1 - z_a}{2} \right) \right], \quad (\text{B.1})$$

where: the sum goes over the whole SM spectrum from massless states to the QCD pseudo-Goldstone bosons (after which the theory becomes non-perturbative),  $F_a = 0, 1$  if the  $a$ -th state is bosonic or fermionic respectively,  $n_a$  counts the d.o.f. of the  $a$ -th state (1 for scalars, 2 for massless vectors, 4 for Dirac fermions...),  $m_a$  is the mass,  $q_a$  is the absolute value of the electric charge normalized to that of the electron  $e$ ,  $A_\phi$  is the Wilson loop modulus and  $z_a = 0, 1$  for periodic or antiperiodic boundary conditions.

Because of the asymptotic behavior of  $V_C$  (eq. (A.5)), as long as  $R$  is away from threshold regions ( $\sim 1/m_a$ ) the total contribution to  $V$  is just the sum of the massless contributions from states that are lighter than  $1/R$ . All these contributions are the same up to a constant factor that depend on the number of d.o.f., the periodicity of the field (also due to a non-trivial Wilson loop) and on the fermionic number of the state ( $F_a$ ). Just looking at these factors one can check the overall sign of the contribution for each  $R$ , which determines the derivative of  $V$  with respect to  $R$ , thus the presence of stationary points. In table 1 we reported such counting for periodic boundary conditions, which shows that besides the neutrino vacuum and a saddle point at the electron scale no other stationary points show up until  $R \approx \Lambda_{\text{QCD}}^{-1}$ .

At this point the perturbation theory breaks down and we cannot trust the formula for the potential (B.1) anymore. In order to study the radion potential around the QCD scale one would need a non-perturbative analysis, using, for instance, lattice QCD simulations. So at the moment we cannot say whether other SM vacua are present in this region for the radion. However, at smaller distances, the strong interaction becomes weak and we can restart using perturbative formulae for our study. This times counting the elementary d.o.f.: gluons, quarks...

At this point, however, the structure of the effective potential gets much more involved. First of all, quarks bring fractional charges that, at fixed radius, produce more than one local minima for the Wilson loop. Second, also gluons can develop non-trivial Wilson loops. There are actually two more moduli ( $G_\phi^{(1,2)}$ ) to be considered, associated to the generators of the Cartan subalgebra of  $SU(3)$ . Both quarks and gluons generate, at the quantum level, non-trivial contributions to the scalar potential for these two fields. If one, or both of them, develop a non-vanishing expectation value than the  $SU(3)$  color group breaks spontaneously into  $SU(2) \times U(1)$  or  $U(1) \times U(1)$ . The effective potential now read

$$V = \frac{2\pi r^3 \Lambda_4}{R^2} + \sum_a (-1)^{F_a} n_a V_a,$$

where for gluons

$$V_a = 2V_C^{(1)} [R, 0, 0] + 2V_C^{(1)} \left[ R, 0, 2\pi \left( G_\phi^{(1)} - G_\phi^{(2)} \right) \right] \\ + 2V_C^{(1)} \left[ R, m_a, 2\pi \left( 2G_\phi^{(1)} + G_\phi^{(2)} \right) \right] + 2V_C^{(1)} \left[ R, m_a, 2\pi \left( G_\phi^{(1)} + 2G_\phi^{(2)} \right) \right],$$

while for the other fields:

$$V_a = V_C^{(1)} \left[ R, m_a, 2\pi \left( q_a A_\phi + g G_\phi^{(1)} + \frac{1-z_a}{2} \right) \right] \\ + g V_C^{(1)} \left[ R, m_a, 2\pi \left( q_a A_\phi + g G_\phi^{(2)} + \frac{1-z_a}{2} \right) \right] \\ + g V_C^{(1)} \left[ R, m_a, 2\pi \left( q_a A_\phi - g G_\phi^{(1)} - g G_\phi^{(2)} + \frac{1-z_a}{2} \right) \right],$$

where  $g = 1$  (or  $0$ ) if the field is (or is not) a quark. The potential became a highly non-trivial function of the radion and the three Wilson loops ( $A_\phi, G_\phi^{(1,2)}$ ) and the search for stationary points becomes much more involved.

Above the weak scale one would need to know also the details of the electro-weak symmetry breaking sector and, eventually, of its extension, as well as to take into account the effects from the Wilson loop of the weak, and eventually others, gauge bosons.

## B.2 3D vacua in standard model extensions

Until now we restricted our discussion to the bare Standard Model action, dressed up just with General Relativity, cosmological constant and neutrino masses. If there are new light d.o.f., which for any reasons escaped direct and indirect search, the structure of the vacua may change dramatically. Let us rapidly discuss some of the possibilities. Clearly sterile neutrinos, light scalars interacting gravitationally or vector fields with very small couplings would have important effects on the analysis. String theory, and in general extra-dimensional theories, usually produce, after the stabilization of the moduli, a plethora of light scalar fields, which interacts mainly gravitationally. The presence of such fields may alter the form of the radion potential, removing, for instance, the neutrino minima and/or creating new minima at higher scales.

More interesting would be the presence of an axion “ $a$ ”. Indeed, besides the usual Casimir contribution, its shift symmetry could be used to switch on a flux for its field strength along the compact dimension

$$\oint_{S_1} da = f.$$

In this way the following extra contribution to the effective potential would arise

$$\frac{f^2 r^3}{4\pi R^4}.$$

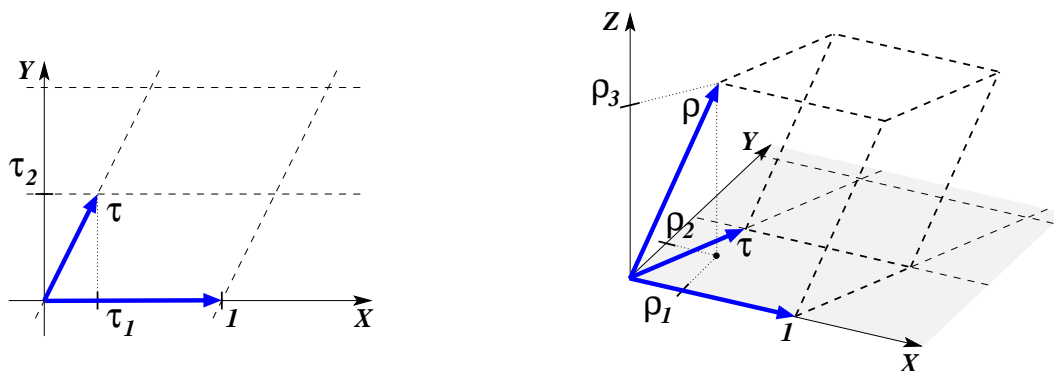
In general the shift symmetry of the axion is broken to a discrete subgroup, which quantizes the flux  $f = n f_a$  in units of the axion decay constant  $f_a$ . For the QCD axion, if it exists, cosmological bounds set  $f_a \sim 10^9 \div 10^{12}$  GeV (see e.g [2]). In this case a non-trivial flux would wipe out all Casimir vacua with  $R_0 \gtrsim f_a^{-1}$ , while vacua with  $R_0$  smaller than this scale will remain because Casimir energy dominates over the flux in this region. Moreover, each of the surviving vacuum will be replicated  $n$  times, with  $n \sim 1/(R_0 f_a)$ , each of the replica with a different flux label. Besides the fluxes, also the Casimir contribution from the QCD axions can lead to interesting consequences. Notice indeed that the actual limits on the QCD axion mass are  $m_a \sim 10^{-6} \div 10^{-2}$  eV [2], right on the neutrino-vacuum scale. The presence of an axion in this range would increase the probability to find a vacuum also in the case of Dirac neutrino. Indeed with normal hierarchy the bounds on the lightest Dirac neutrino in order to have a local minimum are weakened to  $m_{\nu_1} \gtrsim (4.85 \div 6.4) \cdot 10^{-3}$  eV, while with inverted hierarchy an AdS minimum would always exist, also in the Dirac case.

Another possible source of modification of the effective radion potential could be supersymmetry (SUSY). At low energy a light goldstino or gravitino would clearly affect the structure of the minima. The fact that at high energies the fermionic and bosonic d.o.f. are the same because of SUSY suggests the possibility to have new vacua at the SUSY-breaking scale. Moreover at higher energies all the contributions to the Wilson loop would disappear again leaving a number of approximate moduli in the effective theory.

Finally, even without going out of the SM there are other ingredients that can be used to find other vacua, like modifying the boundary conditions with discrete and/or continuous global symmetries like the fermionic  $\mathbb{Z}_2$  symmetry,  $B - L \dots$  or by considering compactifications on more than one dimension. The latter possibility will be explored in the next sections. As we will see the analysis will be a little subtler than in the case of toroidal compactifications in higher-dimensional models, for in our low-dimensional setups several degrees of freedom will not be dynamical. We want to see what is the analogue of looking for minima in the radion potential for finding (meta)stable vacua.

### B.3 2D SM vacua

Let us start by compactifying two spatial dimensions on a two-torus. The torus can be parameterized as usual by the area  $A$  and by the complex modulus  $\tau = \tau_1 + i\tau_2$ , see figure 9.



**Figure 9:** *Left:* Standard parameterization of a two-torus. The spatial metric is flat in the  $X$ - $Y$  plane. The torus is defined by identifying points that differ by integer combinations of the two blue vectors,  $(1, 0)$  and  $(\tau_1, \tau_2)$ . *Right:* Adding a third dimension  $Z$  and another vector  $\rho$  gives an analogous parameterization for a three-torus.

Calling  $X^2$  and  $X^3$  the compact coordinates with periodicity 1,  $X^i \sim X^i + 1$ , the metric takes the form [18]

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta + A \gamma_{ij} dX^i dX^j, \tag{B.2}$$

with  $i$  and  $j$  labeling the compact directions,  $\alpha$  and  $\beta$  labeling the non-compact directions, and

$$\gamma_{ij} = \frac{1}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}. \tag{B.3}$$

Notice that  $\gamma_{ij}$  has unit determinant, so  $A$  really measures the area of the torus. For simplicity we are not including the “graviphotons”  $g_{\alpha 2}$  and  $g_{\alpha 3}$  in the metric above — their inclusion would not qualitatively change the picture.

We dimensionally reduce the 4D Einstein-Hilbert action by imposing that the fields  $g_{\alpha\beta}$ ,  $A$ , and  $\tau$  only depend on the non-compact coordinates  $(t, x)$ . We get

$$S = \int d^2x \sqrt{-g_{(2)}} \left[ \frac{1}{2} M_4^2 (A \mathcal{R}_{(2)} + \frac{A}{2\tau_2^2} |\partial_\alpha \tau|^2) - V(A, \tau) \right], \tag{B.4}$$

where we included a potential energy for  $A$  and  $\tau$ , coming from diverse sources like those studied throughout the paper.

Notice that if we imagine starting from 3D rather than 4D and compactifying one dimension on a circle, we end up with the same action as above for the radion field  $R$ ,

$$S = \int d^2x \sqrt{-g_{(2)}} \left[ \frac{1}{2} M_3 R \mathcal{R}_{(2)} - V(R) \right], \tag{B.5}$$

and obviously no  $\tau$  degrees of freedom. Therefore everything we say in this section is readily exportable to this case as well, and in particular it applies to the  $3D \rightarrow 2D$  interpolations discussed in section 4.

**General analysis.** Like the radion in a compactification from 4D to 3D,  $A$  does not have a kinetic term on its own. But unlike in the radion case, we now cannot demix  $A$  from the 2D metric by means of a suitable conformal transformation. To see this and in order to get some intuition on the dynamics of the system, it is instructive to work in  $D = 2 + \epsilon$  dimensions. Then the needed conformal transformation for going to Einstein frame is

$$g_{\alpha\beta} = A^{-2/\epsilon} \hat{g}_{\alpha\beta} . \tag{B.6}$$

This demixes  $A$  from the metric and generates a kinetic term for  $A$ . The action becomes

$$S_{2+\epsilon} = \int d^2x \sqrt{-\hat{g}_{(2+\epsilon)}} \left[ \hat{\mathcal{R}}_{(2+\epsilon)} + \frac{1}{\epsilon} \frac{(\partial_\alpha A)^2}{A^2} + \frac{|\partial_\alpha \tau|^2}{2\tau_2^2} - A^{-(2+\epsilon)/\epsilon} V(A, \tau) \right] , \tag{B.7}$$

where for notational convenience we set  $\frac{1}{2}M_4^2 = 1$ . This procedure is obviously singular for  $\epsilon \rightarrow 0$ , but the divergence of the  $A$  kinetic term suggests that in  $D = 2$  fluctuations of  $A$  are decoupled. Indeed the canonically normalized area field  $\phi = \frac{1}{\sqrt{\epsilon}} \log A/A_0$  becomes a free field when we send  $\epsilon$  to zero,

$$\mathcal{L}_\phi = (\partial\phi)^2 - A_0^{-\frac{2+\epsilon}{\epsilon}} e^{-\frac{2+\epsilon}{\sqrt{\epsilon}}\phi} \cdot V(e^{\sqrt{\epsilon}\phi}, \tau) \rightarrow (\partial\phi)^2 , \tag{B.8}$$

and the corresponding fluctuations in the area vanish,  $A = A_0 e^{\sqrt{\epsilon}\phi} \rightarrow A_0$ .

The ‘vacua’ are the minima of the effective potential  $V_{\text{eff}} = A^{-(2+\epsilon)/\epsilon} V$ , which correspond to points where  $\partial_{\tau_{1,2}} V = 0$  and

$$\frac{2+\epsilon}{\epsilon} V = A \partial_A V . \tag{B.9}$$

In the  $\epsilon \rightarrow 0$  limit vacua are characterized by a vanishing  $V$ . Einstein’s equations give us the curvature of these vacua,

$$\hat{G}_{\alpha\beta} = -V_{\text{eff}} g_{\alpha\beta} \quad \Rightarrow \quad \hat{\mathcal{R}}_{(2+\epsilon)} = \frac{2+\epsilon}{\epsilon} A^{-(2+\epsilon)/\epsilon} V = A^{-2/\epsilon} \partial_A V . \tag{B.10}$$

Although the Einstein-frame curvature goes to zero for vanishing  $\epsilon$ , the curvature in the original conformal frame is finite,  $\mathcal{R}_{(2)} = \partial_A V$ . So it is  $\partial_A V$  rather than  $V$  itself that plays the role of a cosmological constant.

Notice that for small but finite  $\epsilon$  the curvature of the effective potential on vacuum solutions is

$$\partial_A^2 V_{\text{eff}} = A^{-(2+\epsilon)/\epsilon} \left[ -\frac{(6+2\epsilon)}{\epsilon} \frac{\partial_A V}{A} + \partial_A^2 V \right] \simeq -\frac{6}{\epsilon} A^{-2/\epsilon} \cdot \partial_A V . \tag{B.11}$$

Since de Sitter vacua correspond to positive  $\partial_A V$  while Anti-de Sitter ones have negative  $\partial_A V$ , we conclude that the former sit at maxima of the effective potential and are therefore unstable, while the latter are stable minima. This is to be contrasted with the higher-dimensional cases, where dS/AdS vacua can be either local maxima or minima. Of course when we send  $\epsilon$  to zero the area  $A$  decouples — as we saw the canonically normalized field sees no potential at all in the  $D = 2$  limit — and the de Sitter vacua are stable as well. Still this could be the formal reason why in interpolating from 3D to dS<sub>2</sub> one ends up

with solutions that have the interpretation of “top of the hill” inflation, as we found in section 4.6.

The analysis we just sketched gives indeed the correct results, as we will now see by analyzing directly the action (B.4). We first want to characterize the vacua. Vacuum solutions are solutions in which the moduli  $A$  and  $\tau$  have constant vev’s, so we set to zero all gradients in the field equations.  $A$  appears in the action (B.4) as a Lagrange multiplier. Variation with respect to  $A$  yields the equation

$$\mathcal{R}_{(2)} = \partial_A V, \tag{B.12}$$

which determines  $\mathcal{R}_{(2)}$ . The metric too is non-dynamical in 2D. In fact the variation of  $\mathcal{R}_{(2)}$  is a total derivative, and Einstein’s equations are thus a constraint on the matter sector,

$$V = 0. \tag{B.13}$$

Finally, variation with respect to  $\tau_{1,2}$  yields a standard stationarity condition,

$$\partial_{\tau_{1,2}} V = 0. \tag{B.14}$$

We thus see that 2D vacua are points in the  $A$ - $\tau$  space in which  $V$  vanishes and is stationary with respect to  $\tau$ . Then  $\partial_A V$  determines the effective two-dimensional c.c., through eq. (B.12).

We now study the stability of such vacua against small fluctuations. For simplicity we consider just the Minkowski case, that is we assume  $\partial_A V = 0$  on the vacuum solution. The analysis can be easily extended to the dS and AdS cases. We perturb the vacuum with small fluctuations  $h_{\alpha\beta}$ ,  $\delta A$ ,  $\delta\tau_{1,2}$ . The linearized field equations coming from varying the action with respect to  $A$ ,  $g_{\alpha\beta}$ , and  $\tau$  read respectively

$$\delta\mathcal{R}_{(2)} - m_{AA}^2 \delta A - m_{A\tau_a}^2 \delta\tau_a = 0 \tag{B.15}$$

$$\partial_\alpha \partial_\beta \delta A - \eta_{\alpha\beta} \square \delta A = 0 \tag{B.16}$$

$$-c \square \delta\tau_a - m_{\tau_a A}^2 \delta A - m_{\tau_a \tau_b}^2 \delta\tau_b = 0 \tag{B.17}$$

where the mass matrix  $m^2$  is given by  $m_{\phi_i \phi_j}^2 = \partial_{\phi_i} \partial_{\phi_j} V$ ,  $c$  is the combination  $A/\tau_2^2$  evaluated on the vacuum, and we made use of the vacuum equations above. Also all contractions are done by means of the background metric  $\eta_{\alpha\beta}$ . Eq. (B.15) is not a propagation equation for the metric, but rather it is a constraint. This is because in two dimensions the Ricci and Riemann tensors are both determined by the Ricci scalar  $\mathcal{R}_{(2)}$ , with the appropriate tensor structures given by the metric. Thus the only invariant quantity is  $\mathcal{R}_{(2)}$ , and knowing  $\mathcal{R}_{(2)}$  uniquely determines the metric up to gauge transformations. Therefore eq. (B.15) fixes the metric as a function of the other fields. Eq. (B.16) is not dynamical either. In fact its trace imposes  $\square \delta A = 0$ , which plugged back into eq. (B.16) itself gives

$$\partial_\alpha \partial_\beta \delta A = 0. \tag{B.18}$$

This constrains  $\delta A$  to be a linear function of  $x^\alpha$ ; in particular no localized perturbation can be given as an initial condition for  $\delta A$ . In studying the stability of the system against

local perturbations we thus have to set  $\delta A = 0$ . We are then left with eq. (B.17) with  $\delta A = 0$ , which describes a stable system if and only if the mass matrix  $m_{\tau_a \tau_b}^2$  is positive definite. In conclusion, in order for a 2D vacuum to be stable it must be a minimum of the potential along the  $\tau_1, \tau_2$  directions.

**The real world.** So far our discussion has been completely general. We now consider the case of the Standard Model with a small cosmological constant  $\Lambda_4$ . Minimally, the two-dimensional potential has a positive contribution from the c.c.,

$$V_\Lambda = \Lambda_4 A, \tag{B.19}$$

as well as negative and positive contributions from the Casimir energy of bosons and fermions, respectively. The computation of the Casimir energy on the torus proceeds analogously to the cylinder case of appendix A: one writes the two-point function as a sum over images and subtracts the UV divergent part; then the Casimir  $T_{\mu\nu}$  is just given by proper derivatives of the resulting Green's function in the limit where the two points are brought together. The result in 4D for a massless boson is of the form (see e.g. ref. [18])

$$\rho_{\text{Casimir}} \propto -\frac{1}{A^2} \sum'_{n,m} \frac{1}{|n - m\tau|^4}, \tag{B.20}$$

where the primed sum extends from  $-\infty$  to  $+\infty$  excluding the case  $(n, m) = (0, 0)$ . After dimensional reduction this gives a contribution to the two-dimensional potential  $V_{\text{Casimir}} = \rho_{\text{Casimir}} A$ . For massless fermions the result is the same, apart from the overall sign which is positive. For massive particles the result is obviously much more complicated, and we cannot simply model it with a step function like in the cylinder case: for instance for a square torus we could say that a massive particle does not contribute to the Casimir energy until the area  $A$  drops below  $1/m^2$ , and after that it contributes like a massless particle, but for a general torus the combined dependence of this threshold on  $A$  and  $\tau$  will be more involved.

We will not attempt here a detailed analysis of the Casimir energy in the SM as a function of the torus moduli in order to find stable 2D vacua. There is however a simple situation that we can readily study. Suppose that starting from 4D we first compactify  $z$  stabilizing the radius at  $R_z \sim 1$  mm, thanks to the interplay between the c.c. and the Casimir energy as described in section 2. Then we compactify another dimension, say  $y$ , on a much larger circle, so that we can consistently use the 3D effective theory. We want to see if in this situation we can find a stable vacuum. In the 3D theory we have a cosmological constant  $\Lambda_3$  and, among other things, a massless photon. We can then turn on a constant electric field along the (non-compact)  $x$  direction,  $E = F_{0x}$ ; alternately we can turn on an electric field for the graviphoton. Such an electric field does not break 2D Lorentz invariance in the non-compact dimensions, since  $F_{\alpha\beta} \propto \epsilon_{\alpha\beta}$  is Lorentz-invariant. Equivalently, in 3D a 1-form is dual to a scalar  $\phi$ , and a constant electric field along  $x$  corresponds to a constant  $\partial_y \phi = E$ . So the case we are studying is technically the lower dimensional analogue of the axion wrapped around the circle of section B.2. The electric field gives a positive contribution to the 2D energy density that scales like  $f^2/R_z R_y$ , where

$R_y$  is the physical radius of the  $y$  dimension and  $f = E \cdot R_z R_y$  is the conserved flux. Then for large  $R_y$  and  $f$  the Casimir energy coming from the compactness of  $y$  is completely negligible with respect to this classical contribution to the potential.

The full 2D potential we consider is therefore

$$V = \Lambda_3 R_y + \frac{f^2}{R_z R_y} . \tag{B.21}$$

Recall that 2D vacua are characterized by an overall vanishing potential energy. Therefore in the presence of a flux  $f$  we only get a vacuum if the 3D cosmological constant is negative, which could well be the case for the SM as we argued in section 2. In this case the radius is given by

$$R_y = \frac{f}{\sqrt{R_z |\Lambda_3|}} = R_z \frac{f}{\varepsilon} , \tag{B.22}$$

where we defined the quantity  $\varepsilon = \sqrt{R_z^3 |\Lambda_3|}$ . Without fine-tunings of the neutrino masses we expect  $\varepsilon$  to be of order one. However as we will soon see for our approximations to be self-consistent we will have to assume  $\varepsilon \ll 1$ . The curvature of this vacuum is determined by  $\frac{2}{M_3} \partial_{R_y} V$ , where the 3D Planck mass is  $M_3 = R_z \cdot M_4^2$ ; we thus get

$$\mathcal{R}_{(2)} = \frac{2}{M_4^2} \frac{1}{R_z} \partial_{R_y} V = -\frac{4}{M_4^2} \frac{\varepsilon^2}{R_z^4} \tag{B.23}$$

independent of  $f$ . The flux  $f$  is quantized in units of the 4D electric charge  $e$ , so we have a *discretum* of different  $\text{AdS}_2$  vacua parameterized by  $f$ , all with exactly the same curvature radius, which is roughly  $1/\varepsilon$  times larger than our Hubble scale.

For our approximations to be self-consistent we first have to assume that  $R_y \gg R_z$ , which requires  $f \gg \varepsilon$ . Then we have to impose that the electric field does not destabilize the 3D radion,  $R_z$ : after all  $R_z$  was stabilized thanks to the Casimir energy, which is a small quantum effect.  $R_z$  is still stable if the “force”  $\partial_{R_z} V$  is smaller than the typical curvature scale of the stabilizing potential for  $R_z$ , so that the electric flux only moves  $R_z$  slightly away from the minimum. Notice that  $\Lambda_3$  is implicitly a function of  $R_z$ , but by assumption  $R_z$  is at a minimum of  $\Lambda_3$ , so we get no force from that piece of the potential. We have

$$\partial_{R_z} V = -\frac{f^2}{R_z^2 R_y} = -\frac{f \varepsilon}{R_z^3} , \tag{B.24}$$

to be compared with  $\sim 1/R_z^3$ . We thus have to impose  $f \ll 1/\varepsilon$ , which combined with the previous requirement,  $f \gg \varepsilon$ , tells us that our approximations are self consistent only if  $\varepsilon$  is much smaller than one, i.e. if the 3D cosmological constant is unnaturally small. Therefore in general we don’t expect these very asymmetric compactifications to give rise to stable vacua — one should instead consider more symmetrically shaped tori, for which a full analysis of the Casimir energy as a function of the torus moduli is necessary. However if the required fine-tuning is fortuitously realized in the real world and  $\varepsilon$  is actually very small, then there exist  $N \sim 1/(\varepsilon e)$  two-dimensional AdS vacua, parameterized by  $f$ , all with the same 2D curvature length.



### B.4 No 1D “vacua”

We now imagine to compactify all three spatial dimensions on a three-torus. For the discussion that follows the parameterization we choose for the torus is not important, but for concreteness let us parameterize it with the overall (dimensionful) scale factor  $a$  and the shape moduli  $\Phi = (\tau_1, \tau_2, \rho_1, \rho_2, \rho_3)$  defined in figure 9. Then if the compact directions have periodicity 1, the 4D metric reads

$$ds^2 = -N^2 dt^2 + a^2 \gamma_{ij} dX^i dX^j, \tag{B.25}$$

with

$$\gamma_{ij} = \frac{1}{(\rho_3 \tau_2)^{2/3}} \begin{pmatrix} 1 & \tau_1 & \rho_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 & \rho_1 \tau_1 + \rho_2 \tau_2 \\ \rho_1 & \rho_1 \tau_1 + \rho_2 \tau_2 & \rho_1^2 + \rho_2^2 + \rho_3^2 \end{pmatrix}. \tag{B.26}$$

Notice that  $\det \gamma_{ij} = 1$  as before, so the volume of the torus is  $a^3$ . Dimensional reduction yields the 1D action

$$S = \int dt \frac{1}{2} M_4^2 \left[ -\frac{6 \dot{a}^2 a}{N} + \frac{a^3}{N} \dot{\Phi} \cdot K(\Phi) \dot{\Phi} \right] - N V(a, \Phi), \tag{B.27}$$

where  $V$  is the sum of the 4D cosmological constant, Casimir energy density, and possibly other sources of potential energy, all multiplied by the volume of the three-torus, and  $K(\Phi)$  is a  $5 \times 5$  matrix that depends on the shape of the torus and whose explicit form we spare the reader. Its positivity can be readily checked for very symmetric configurations, like the rectangular torus  $\tau_1 = \rho_1 = \rho_2 = 0$ . More generically, the  $\Phi$ 's parameterize the coset manifold  $SL(3)/SO(3)$ , and being  $SO(3)$  the maximal compact subgroup of  $SL(3)$ , the corresponding non-linear sigma model has positive definite kinetic energy.

Generically the action (B.27) describes a cosmology.  $N$  appears as a Lagrange multiplier and its equation of motion is the Hamiltonian constraint  $\mathcal{H} = 0$ , which is nothing but the Friedman equation

$$\mathcal{H} = \frac{1}{2} M_4^2 \left[ -6 \dot{a}^2 a + a^3 \dot{\Phi} \cdot K(\Phi) \dot{\Phi} \right] + V(a, \Phi) = 0, \tag{B.28}$$

where we fixed the gauge  $N = 1$ . We can equivalently set  $N = 1$  directly in the Lagrangian,

$$\mathcal{L} = \frac{1}{2} M_4^2 \left[ -6 \dot{a}^2 a + a^3 \dot{\Phi} \cdot K(\Phi) \dot{\Phi} \right] - V(a, \Phi), \tag{B.29}$$

and supplement the system by the constraint that the total Hamiltonian vanishes, eq. (B.28). Since the Hamiltonian is conserved on the equations of motion of  $\mathcal{L}$ , this is just a constraint on the initial conditions. Notice that  $a$  enters the action with negative kinetic energy.

Usually cosmological solutions evolve with time. In our case this time evolution would correspond to a decompactification, or to a big crunch, or to some anisotropic Kasner-like solution. Instead we are looking for vacua — i.e. static solutions in which the moduli are stabilized. It is evident from the Lagrangian above that a static solution must be an

extremum of the potential, but then to have zero total energy the potential itself should vanish at the same point. So the existence of a truly static solution requires a perfectly tuned potential.

More realistically, in our case we expect  $V$  to develop non trivial features at the micron scale with typical energies of order  $\mu\text{m}^{-1}$ . So let us assume that there is a stationary point of  $V$  at  $a_0 \sim \mu\text{m}$ ,  $\Phi_0 \sim 1$ , but for reasons that will soon become clear let's assume that the potential itself at the stationary point is somewhat smaller than the typical energy scale:  $V_0 \sim \varepsilon^2 \mu\text{m}^{-1}$  with  $\varepsilon \ll 1$ . Then in a neighborhood of  $(a_0, \Phi_0)$  we can expand the Lagrangian at second order in the displacements  $\delta a, \delta\Phi$ . The resulting quadratic Lagrangian describes a set of harmonic oscillators, provided that the Hessian of the potential — the ‘mass matrix’ — have the right signature. In particular, since the  $\Phi$ 's have a positive definite kinetic energy while that of  $a$  is negative,  $V$  should be positively curved along the  $\Phi$  directions and negatively curved along  $a$ .<sup>2</sup> If these conditions are met, then the typical oscillation frequency in all directions is of order of our present Hubble rate  $H_0$ —assuming that the curvature scales of  $V$  are of order  $\mu\text{m}^{-1}$ .

The Hamiltonian constraint fixes the initial oscillation amplitudes. The total energy of the oscillators should vanish, taking into account also the offset  $V_0 \sim \varepsilon^2 \mu\text{m}^{-1}$ . So the typical amplitudes are  $\delta a \sim \varepsilon \mu\text{m}$ ,  $\delta\Phi \sim \varepsilon$ . If  $\varepsilon$  is small the oscillations are small compared to the typical variation scales of the potential, and the perturbative analysis we are sketching here is justified. In this case we have an almost static micron-sized universe that undergoes small periodic oscillations in size and shape on a timescale of order  $10^{10}$  years! Note that a classical description of this motion is justified since the amplitudes of oscillation are much larger than the quantum uncertainties, with  $(\delta_{\text{quantum}} a / \delta_{\text{classical}} a)^2 \sim \mu\text{m} \times H_0$ .

Of course the fact that  $\delta a$  has negative energies signals that the system is unstable once interactions between the two sectors — the “inverted” oscillator and the normal ones — are taken into account. The two sectors can start exciting each other while keeping the total energy fixed, and this happens classically already at perturbative level. However unlike in relativistic field theories with ghosts where the rate of such instability is formally infinite because of Lorentz symmetry, here the instability is slow and its rate can be reliably computed in perturbation theory.

Despite the appearance of the Planck scale in front of the kinetic terms, the only suppression of interactions in our case comes from the smallness of  $\varepsilon$ —i.e. interactions are not Planck-suppressed. This is because we are studying large classical oscillations, much larger than the typical quantum spread of the ground-state wave-function. Then in the Lagrangian (B.29) we can reabsorb  $M_4$  into a redefinition of time. This only changes the overall normalization of the action, which classically is arbitrary. With this redefinition, there is no small parameter in the Lagrangian, and the importance of interactions is only controlled by the oscillation amplitude,  $\varepsilon$ . Therefore the instability rate is suppressed with

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<sup>2</sup>This is true if the mixed second derivatives  $\partial_a \partial_\Phi V$  are negligible. The general condition for having only oscillatory solutions is that the Hessian of  $V$  with respect to  $\Phi$  and to  $i \cdot a$  be positive definite.

respect to the oscillation frequency by positive powers of  $\varepsilon$ ,

$$\Gamma \sim H_0 [\varepsilon + \varepsilon^2 + \dots]. \quad (\text{B.30})$$

It is easy to convince one's self that the leading term is there only if resonance phenomena are possible, i.e. if two frequencies are tuned to be equal. Barring this possibility, the instability rate is generically of order  $\Gamma \sim \varepsilon^2 H_0$ . A detailed analysis of the classical dynamics of two coupled harmonic oscillators, one of which has negative energy, confirms this quick estimate.

In conclusion, if the potential energy at the stationary point is much smaller than the typical energy scales and the mass matrix has the right signature, then there exists a micron sized solution that slightly oscillates in size and shape with a period of order  $H_0^{-1}$ . Eventually it is unstable against decompactification or crunching, but on a longer timescale of order  $H_0^{-1}/\varepsilon^2$ . If instead there are no special tunings in the potential, then the instability time is of order of the would-be oscillation frequency and there is no conceptual difference between the situation we are describing and a standard cosmological solution.

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